How Do We Know Anything about Mathematics? - A Defence of Platonism

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ABSTRACT
In this paper I will try to say something about how we can know anything about mathematics from a Platonist's point of view.
For Platonism the problem of how we know things about mathematics seems to be particularly acute. So, I will firstly present the epistemological problem and secondly see how it could be solved.
I will not try to argue that Platonism is true; I will just try to protect Platonism from what is regarded to be the best attack on Platonism - Benacerraf's epistemological attack.

Key words: mathematical Platonism; epistemology; Benacerraf's epistemological argument.

1. Introduction.

Mathematical Platonism is the view that there exist (at least) enough abstract objects to make true at face value the bulk of mathematical statements we accept.

Mathematical Platonism is the view that - at least some - mathematical objects (such as numbers, sets, and functions) exist, independently of our constructions and beliefs; and that the mathematical statements we take to be true are, by and large, true (and apt for a strong notion of truth).

The main epistemological problem for Platonism was familiar even to Plato. But its clearest and most prominent formulation in the contemporary philosophy of mathematics is that of Paul Benacerraf[1].

[1] In his (1973) 'Mathematical Truth', Journal of Philosophy **70**, pp. 661-679.

It can be formulated briefly in the following way: if the causal theory of knowledge is true and mathematical objects are abstract and therefore causally inert, then no mathematical knowledge is possible. The obvious conclusion to be drawn, since we do have some mathematical knowledge, is that Platonism is untenable.

2. The epistemological argument.

The epistemological argument against Platonism, as formulated by Benacerraf, is a powerful one. Platonists have to concede that abstract objects are causally inert. Hence, if some causal link with an object of knowledge were a necessary condition of knowledge, as the causal theory of knowledge supposes, mathematical knowledge Platonistically conceived would be impossible. And that would be enough to scotch Platonism.

But is the causal theory of knowledge correct? There are examples[2] in which a knower bears a kind of convoluted and very indirect causal relationship with the object of knowledge and in which it is clear that it is very difficult to accommodate the (even modified) causal theory of knowledge.

What I am saying at this point is that it is wrong to think that the causal theory of knowledge shows that Platonism makes mathematical knowledge impossible. What does it prove? We must not overemphasise the conclusion at this point.

But the question still remains: what does mathematical knowledge, as Platonistically conceived, consist in? or, even, how is it possible? So, what answer should be given to the question: how do we know anything about mathematics?

Now, Platonism is not homogeneous and different versions of it arise according to how the problem of grasping mathematical objects is viewed, and which mathematical objects are held to exist. Some Platonists endorse the view that there is a platonistic intuition that allows us to grasp the basic mathematical objects and theorems. Hardy[3], for example, thinks that we actually "see" certain mathematical results in basically the same way in which a geologist sees a mountain. Similarly, Gödel[4] thinks that there must be a centre responsible for the perception of sets

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[2] For example, those examples concerning future events: If I, e.g., flip a detonator, on which there is an uninterruptible ten second delay, then I *know* there will be an explosion. The explosion is future though: it therefore has no causal effect upon me, mediated or otherwise.
Another sort of case that is more difficult for even the modified causal theory to accommodate is provided by quantum mechanics; for more details see Brown (1999), pp. 15-18.

[3] See Hardy (1948), pp. 24-34.

[4] See his (1944) 'Russell's mathematical logic', repr. in Benacerraf, Putnam (eds.) (1983), pp. 447-69.

located near the neuronic speech centre, and that we grasp sets with a perception analogous to sense perception. Logicists and neo-logicists[5] deny the existence of the kind of intuition Gödel had in mind and claim that our mathematical knowledge is based on our capacity to grasp mathematical objects by the specifically reasoning faculties of the mind.

I consider that a satisfactory epistemology for Platonism is already evident in the considerations on which the attraction of Platonism largely rests.

And we arrive at an answer more easily if we first examine another problem that seems to be particularly problematic for Platonism, viz. the problem of applied mathematics. So, let us first have a closer look at it. The "application" problem arises from the fact that mathematics as a whole, and in particular arithmetic and analysis, are applicable to the physical, empirically perceptible world not just in the sense of being true, but also in the sense of being useful. This problem is hard enough for any philosophy of mathematics to solve. But Platonism has seemed to many to make the problem not just hard, but insoluble. Platonists maintain that mathematical objects are abstract, and hence causally inert and without spatio-temporal location. But how could knowledge of objects of this kind be of any use in the attempt by natural science to explain and understand the natural, *concrete* world? Thus, there is a tension between the view of Platonism that mathematical objects are abstract, and the obvious fact that common sense, and especially, science, successfully applies mathematics to the physical world.

The application problem is a somewhat ironic turning-of-the-tables on the use to which the realist puts the fact that mathematics plays an ineliminable role in natural science in his "indispensability" argument. According to the argument, the role mathematics plays in science provides strong evidence - and perhaps the best evidence we have - that mathematics is true.

It means that, among our best theories of the world are the theories of empirical science and science (e.g. physics) quantifies (seemingly unavoidably) over mathematical objects. That's why we have good reasons to believe in the existence of mathematical objects, unless and until we can do science without postulating them.

[5] See, e.g., Hale & Wright (2001).

The argument can be formulated in more detail as composed of three premises:

(i) Indispensability: mathematics is indispensable to natural science

(ii) Confirmational holism: if the observational evidence supports a scientific theory, it supports the theoretical apparatus as a whole rather than some particular hypotheses

(iii) Naturalism: natural science is for us the ultimate arbiter of truth and existence.

The argument then goes as follows: for (i), mathematics is indispensable to our theory of the world; by (ii), any evidence we have for the truth of some scientific theory is at the same time evidence for the truth of the mathematical apparatus employed in the formulation of the theory and in the derivation from the theory of the predictions confirmation of which constitutes the evidence for the theory; therefore, by (iii), mathematics is true[6].

However, some realists, mostly Platonists, are opposed to so great an emphasis on the indispensability argument. Their worry is that this argument assimilates mathematics too much to empirical science, and thereby fails to respect its *sui generis* nature. Namely, by definition the indispensability argument can only claim that there is direct evidence for the truth of those mathematical theorems that play an indispensable role in the formulation and manipulation of scientific empirical theories.

But what about the part of mathematics that does not play such a role in science?

No doubt some of it can then be supported indirectly, by means of "abductive" reasoning: mathematical theorems T for which there is no direct evidence are supported indirectly because they follow from axioms which neatly unify and explain those theorems for which science provides direct evidence. But here judgements of "neatness" and "explanatory power" are essentially mathematical, and in any case one might expect some slack to remain, in that some mathematical theorems will fail even to fall into the category of theorems for which there is direct evidence in this sense. But this can be solved in the sense that the argument from obviousness might take up that slack.

But while realism is a large component of the doctrine of Platonism, it now turns out that the very phenomena to which the Platonist might well appeal to

[6] For more details see, e.g., Field (1980), pp. 7-23.

substantiate his realism, undermine the nuance which characterises his specific version of realism.

However, in response to this argument I do not think that Platonism renders the application problem especially acute. If, to take a crude example, "2+2=4" is true, is it not then obvious that whatever two units - cats or missiles or whatever - we take and add two more of them we will get four of them? The relation between what "2+2=4" is true in virtue of, and what 2Fs plus 2 more Fs = 4Fs is true in virtue of, can be expressed as follows: 2Fs+2Fs=4Fs is true in virtue of 2+2 being 4. *That* much is obvious, irrespective of whatever it is in virtue of which 2+2=4. There is simply no question that a platonistic construal of the latter truth makes the truth that 2 concrete Fs plus 2 further concrete Fs makes 4 concrete Fs. Since the sequence 1F, 2F, 3F, ... exemplifies the natural number structure, 2Fs+2Fs=4Fs exemplifies the form "2+2=4". It follows that what makes "2Fs+2Fs=4Fs" true is the fact that "2+2=4" is true. And it does so even if the latter is true in virtue of the properties of independently existing entities.

Now, how is this related to the epistemological problem? I think a satisfactory epistemology for Platonism is already evident in the considerations on which the attractions of Platonism largely rest. As I have just noted, the claim that the problem of the applicability of mathematics to the empirical world is particularly acute for Platonism is somewhat ironic, since it is the fact of the applicability of mathematics to the natural world which provides one of the main grounds for the realism about mathematics which Platonism exemplifies. And now I want to suggest that it is the indispensability of mathematics to natural science - and hence its applicability - that provides Platonism with the epistemology it needs, and hence, which makes mathematical knowledge possible. Let us consider again my arguing that 2Fs+2Fs=4Fs is true in virtue of the fact that 2+2=4. That is the ontological order of things.

What about the epistemological order of things?

One might think that the same 2Fs+2Fs=4Fs amounts to F+F+F+F, and this is like saying 1+1+1+1, given that we treat F as a sort of unit. And then the fact that the latter sum is 4 informs us the former sum is 4Fs. We are able to abstract details and treat, for example, cats as units. We know that sequences of concrete objects can exemplify mathematical structures which permits us to conclude that it is possible to (sort of) apply number equations to equations with concrete objects like 2Fs+2Fs=4Fs.

However Russell expresses a very different viewpoint when he writes:

> The proposition 2+2=4 itself strikes us now as obvious; and if we were asked to prove that 2sheep+2sheep=4sheep, we should be inclined to deduce it from 2+2=4. But the proposition '2sheep+2sheep=4sheep' was probably known to shepherds thousands of years before the proposition 2+2=4 was discovered; and when 2+2=4 was first discovered, it was probably inferred from the case of sheep and other concrete cases[7].

Since we now find 2+2=4 obvious, and deduce that 2sheep+2sheep=4sheep, what are the grounds for Russell's confidence that the latter was known to us before the former? Could the shepherds not have found it obvious that 2+2=4 is true, and then applied this knowledge in order to count sheep?

Well, perhaps. But the indispensability argument suggests that at least in more complicated cases, and perhaps even this one, the basic thrust of Russell's position is correct: the epistemological order of things is the reverse of the ontological one. It is the applicability of mathematics, and in particular its indispensability to science, which gives us most reason to think that the mathematical theorems we take to be true really are true, independently of us and our mathematical practice. Since mathematical knowledge amounts to reasonable, grounded, true belief, and this fact about the role of mathematics in our understanding of the natural world is our primary ground for mathematical belief, it follows that mathematical knowledge is grounded in this fact too. Obviousness might still have something to do with mathematical knowledge, ultimately. Nevertheless, it is the epistemological holism embodied in the argument from indispensability which really vindicates the Platonist conception of mathematical truth, and, hence, knowledge.

[7] Russell (1973), p. 272.

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