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Structuralism in the Philosophy of Mathematics

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Abstract
Structuralism in the philosophy of mathematics is the doctrine according to which mathematics is about structures.
The paper concerns itself with the question of whether or not structuralism, which (at least for some versions) can be treated as a sort of non-traditional Platonism, solves problems that Platonism is concerned with: the problem of indeterminacy and the epistemological problem.
The paper is divided in five parts: the first one-the introduction, the second one is dedicated to different versions of structuralism, the third part deals with the ontology, both of mathematical objects and structures themselves, the fourth part is concerned with the problem of indeterminacy and structuralism’s answer to it and the last one about the epistemological problem for Platonism and the way structuralism tries to solve it. The parts three, four and five contain the author’s attempted objections and criticism. The author tries to show that structuralism does not give satisfactory answers to the problems of Platonism in the philosophy of mathematics

1. Introduction

As Benacerraf emphasises, Platonism in the philosophy of mathematics faces two main problems, one epistemological, the other ontological. The main epistemological problem can be formulated briefly in the following way: if the causal theory of knowledge is true and mathematical objects are abstract and therefore causally inert, then no mathematical knowledge is possible; since we do have some mathematical knowledge, Platonism is unsustainable. The ontological problem is one of apparent indeterminacy: since it is possible to reduce any field of mathematics to set theory we can reduce numbers too; but there are several possible reductions of arithmetic to set theory and the problem is how to determine which one is the right one.
This paper concerns whether or not a recent doctrine known as "structuralism", which (at least for some versions) can be treated as a sort of non-traditional Platonism, solves these two problems.
The paper is divided in five parts: the first one-the introduction, the second one dedicated to the different versions of structuralism, the third part about the ontology both of mathematical objects and structures themselves, the fourth part about the problem of indeterminacy and structuralism’s answer to it and the last one about the epistemological problem for Platonism and the way structuralism tries to solve it. The parts three, four and five are followed by my attempted objections.

2. Versions of structuralism

The basic thesis of structuralism is that mathematics is about structures.

> The mathematics book is not describing a system of sets or Platonic objects or people. It describes a structure or a class of structures.[1]

To appreciate this thesis one needs to understand the distinction between structures and systems: a structure is the abstract form of a system and a system is a collection of (independently existing) objects with certain relations.

According to Dummett, there are two main versions of this doctrine: mystical and hardheaded.[2]

Mystical structuralism comprises two main thesis. First, the idea that mathematics is concerned with abstract structures and that the elements of the structures have no properties beside the structural ones i.e. no non-structural properties. Mathematical objects are structureless places in structures and are not given in isolation. Second, (Dedekind's) view that abstract systems are free creations of the human mind. We create systems by psychological abstraction and we need a non-abstract system to begin with. Dedekind endorses:

> the need to maintain that we can find infinite system of objects - system isomorphic to the natural numbers and others isomorphic to the real numbers - in nature;...[3]

By contrast, according to the hardheaded version, the elements of a system analyzed in mathematics are not mathematical objects since objects can't have just structural properties.

> It is part of such a view that the elements of the systems with which a mathematical theory is concerned are not themselves mathematical objects, but, in a broad sense, empirical ones; it is not the concern of mathematics whether such systems do or do not exist.[4]

A mathematical theory concerns not just one mathematical system but all systems with a given structure. When we talk about a structure that is just a shorthand for talking about all systems that exemplify the structure. It is a sort of structuralism 'without structures'.

Shapiro[5] introduces a slightly different distinction. Dummett's mystical structuralism is *ante rem* structuralism in Shapiro's terms. *Ante rem* structuralism holds that structures are genuine objects and they exist even if there are no systems of objects that exemplify them.[6] It is the view that mathematics is concerned with abstract structures where:

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[1]Shapiro S. *Philosophy of Mathematics - Structure and Ontology*, p.131-2

[2]Dummett M. *Frege - Philosophy of Mathematics*, pp. 295-296

[3]Dummett M. *Frege - Philosophy of Mathematics*, p. 296

[4]Dummett M. *Frege - Philosophy of Mathematics*, p. 296

[5]See Shapiro S. *Philosophy of Mathematics - Structure and Ontology*

[6]Except for the structure itself. Namely, each structure exemplifies itself since its places, bona fide objects, form a system which exemplifies the structure.

> ...a structure is a pattern, the form of a system. ...Thus, structure is to structured as pattern is to patterned, as universal is to subsumed particular, as type is to token.[7]

A structure is an abstract form of a system in which 'any features [of the objects] that do not affect how they are related to other objects in the system'[8] are ignored. Structures are not necessarily mathematical: we can talk about, e.g., a chess configuration or a basketball defence as well as we can talk about the natural number structure.

However, even though Shapiro identifies mystical and *ante rem* structuralism they should be distinguished; for, Shapiro does not endorse the view that abstract system are creations of the human mind.

Shapiro calls Dummett's heardheaded structuralism 'eliminative structuralism'.[9] He identifies eliminative structuralism with what he calls *in re* structuralism even though these two versions do not seem to be identical. *In re* structuralism has an *in re* approach to structures: if all the systems that exemplify the natural number structure disappeared the natural-number structure would disappear too.[10] There is no structure unless there is at least one system that exemplifies it. Since this version has an *in re* approach to structures it faces the difficulty of having a robust background ontology in order to make sense of a vast part of mathematics. According to Shapiro there are three possible solutions to this problem. One possible answer is that there are enough objects for each structure to be exemplified; in order to keep arithmetic from being vacuous it is necessary to assume that there is a system that exemplifies the structure. Shapiro calls this the ontological option.

The second possible option is the already mentioned mystical structuralism.

The third way of solving this difficulty leads to a further version of structuralism (not mentioned in Dummett): modal eliminative structuralism according to which arithmetic is about all logically possible systems of a certain type. So according to modal eliminative structuralism it is not necessary to assume that a system that exemplifies a given structure exists; it suffices that such a system is logically possible. This third option and the *in re* version have not been discussed in this paper.

3. Ontology

Talking about the ontology of structuralism there are two questions that have to be answered: one concerning the ontology of mathematical objects, the other concerning the ontology of structures themselves.

Ante rem structuralism is an ontological realism about mathematical objects:

> Structuralists hold that a nonalgebric field like arithmetic is about a realm of objects - numbers - that exist independently of the mathematician, and they hold that arithmetic assertions have non vacuous, bivalent, objective truth-values in reference to this domain.[11]

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[7]Shapiro S. *Philosophy of Mathematics - Structure and Ontology*, p. 84

[8]Shapiro S. *Philosophy of Mathematics - Structure and Ontology,*, p. 74

[9]Shapiro S. 'Space, number and structure: a tale of two debates', Philosophia Mathematica (3) Vol. 4 (1996), pp. 148-173, page 150

[10]It does not seem for the eliminative structuralist to be committed to such a view, nor the other way round.

[11]Shapiro *Philosophy of Mathematics - Structure and Ontology*, page 72

Ante rem structuralism can't nevertheless be identified with traditional Platonism. According to Platonism it is possible to determine the essence of each number without referring to the other numbers. The problem for the Platonist (non-structuralist) consists in the fact that even though a mathematical theory is about certain entities we can't definitely determine what kind of objects they are. Structuralists reject the Platonist view since

> The essence of a natural number is its *relations* to the other numbers. ... There is no more to the individual numbers "in themselves" than the relations they bear to each other.[12]

What about the ontological status of structures?
Ante re structuralism

> takes a realist approach, holding that structures exist as legitimate objects in their own right. According to this view a given structure exists independently of any system that exemplifies it.' [13]

According to Shapiro structures are genuine objects. Every structure is a universal and every system that exemplifies it is an instance[14]; the properties of structures are independent of us. 'Mathematical assertions are read at face value, and numerals are singular terms'[15].

What are the difficulties connected with this view?
1) According to Shapiro, numbers are bona fide objects as any objects are, '[M]athematical objects - places in structures - are abstract and causally inert.'[16] But at the same time he also endorses the view that the term 'object' is relative to the theory in question:

> Our conclusion is that in mathematics, at least, one should think of "object" as elliptical for "object of a theory".... The idea of a single universe, divided into objects a priori, is rejected here.[17]

As they stand, these two views seem to make the theory inconsistent.
2) In his book *Philosophy of Mathematics - Structure and Ontology*, Shapiro talks about the objective existence of the natural - number structure:

> The natural-number structure has objective existence and facts about it are not of our making.[18]

That means that the natural-number structure exists independently of us and therefore of our linguistic resources, too. On the other hand, this view seems to make questionable the idea that 'the language characterizes or determines a structure (or class of structure) if it characterizes anything at all.'[19]

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[12] Shapiro *Philosophy of Mathematics - Structure and Ontology*, page 72-3
[13]Shapiro S. 'Space, number and structure: a tale of two debates', page 149
[14]When Shapiro uses the term 'universal' he refers to a pattern or structure, the 'particular' refers to a system of related objects rather then to an individual object.
[15]Shapiro S. in the introduction to the volume four (special issue: Mathematical Structuralism) of the Philosophia Mathematica (3) Vol. 4 (1996), pp. 81-82
[16]Shapiro S. *Philosophy of Mathematics - Structure and Ontology*, p. 112
[17]Shapiro S. *Philosophy of Mathematics - Structure and Ontology*, p. 127
[18]See p. 137
[19]Shapiro S. *Philosophy of Mathematics - Structure and Ontology*, p. 131

> The point is that the way humans apprehend structures and the way we "divide" the mathematical universe into structures, systems, and objects depends on our linguistic resources.[20]

This view suggests that the distinction between structures and systems does depend of our language. So, Shapiro seems to be inclined to say that there are no language-independent objects, but also that "the contents of the universe exist independent of us and our linguistic lives" which does not seem to be a consistent view.

3) Shapiro sustains that (*ante rem*) structuralism can't be identified with traditional Platonism because of the Platonist view that natural numbers are also independent of each other but, as a matter of fact, traditional Platonism does not have to be committed to this view.

4. How does structuralism solve the problem of indeterminacy?

Eliminative structuralism (Benacerraf) and *ante rem* structuralism (Shapiro) try to solve the problem of indeterminacy in indifferent ways:

i) Eliminative structuralism. In his article 'What numbers could not be' Benacerraf tries to solve the problem of indeterminacy, i.e. of identification of numbers with some sort of sets by saying that, since to identify the numbers with sets there are different possibilities, numbers can't be sets after all.

We might, for example, identify numbers both with Zermelo's and von Neumann's ordinals (there are as a matter of fact infinitely many possibilities). The problem is acute because there appear to be no argument for settling it; there is no way to determine the truth value of sentences like the identity '2={Ø,{Ø}}'. Benacerraf concludes that there is 'no "correct" account that discriminates among all the accounts satisfying the conditions ...'[21]; the only possible conclusion is therefore that numbers could not be sets at all.

Benacerraf extends the argument for the assertion that numbers can't be sets to the conclusion that numbers are not objects at all. The problem of trying to identify numbers with some sort of objects is, according to Benacerraf, simply pointless:

> The pointlessness of trying to determine which objects the numbers are thus derives directly from the pointlessness of asking the question of any individual number.
>
> ...
>
> Therefore, numbers are not objects at all, because in giving the properties (that is, necessary and sufficient) of numbers you merely characterize an *abstract structure* - and the distinction lies in the fact that the "element" of the structure have no properties other than those relating them to other "elements" of the same structure.[22]

The solution that Benacerraf offers is the view that mathematics is about structures. He adopts 'eliminative' structuralism: numbers are nothing more than places in the 'natural number' structure. To be the number 3 means nothing more than to be preceded by 2,1, 0 and to be followed by 4, 5, and so on; '*any* object can *play the role*

[20]Shapiro S. *Philosophy of Mathematics - Structure and Ontology*, p. 137
[21]Benacerraf P. 'What numbers could not be', Philosophical Review **74** (1965), pp. 47-73, page 281
[22]Benacerraf P. 'What numbers could not be', page 291

of 3'[23], which means that every object can be in the third place i.e. the third element in a progression. Number theory is therefore not about particular objects-the numbers; it is about the properties of all the systems of the order type of the numbers. Benacerraf is, as a matter of fact, denying the existence of numbers, i.e. he identifies numbers with numerals:

> there are not two kinds of things, numbers and numbers words, but just one, the words themselves.[24]
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> . . .
>
> in counting, we do not correlate sets with initial segments of the numbers as extra linguistic entities, but correlate sets with initial segments of the sequence of number *words*.[25]

In this way Benacerraf avoids the question of what kind of objects the number are and therefore questions like the
Frege's 'Caesar Problem' - 'whether any concept has the number Julius Caesar belonging to it, or whether that same familiar conqueror of Gaul is a number or is not.'[26]

It seems that there are two main problems in Benacerraf's theory:
1) the adeguateness of all system with a natural number structure
2) the identification of numbers with numerals
1) This problem arises if we accept that '*any* object can *play the role of* 3' and therefore 'any system of objects, sets or not, that forms a recursive progression must be adequate'[27]?
Let us take the example in which we want to apply numbers to the real world; we need them to determine for example how many letters the word 'hand' has (or how many hands we have). Now, if the only thing that matters is just the structure and if 'any system of objects, whether sets or not, that forms a recursive progression must be adequate'[28] then it should be the same which progression we choose in order to count, i.e. which model of the natural number structure we use. But, we'd rather have <u>four</u> letters in the word 'hand' (as well as <u>two</u> hands) which means that the progression we are looking for is *a specific* one, the one which allows us to have four letters (and exactly two hands) and in which, if we add one more letter, the numbers of letter will be five. There are infinitely many progression unacceptable in that sense: let us take the progression 0, 2, 4, ... which has the natural number structure; if we accepted this one we would have 4 hands and the word 'hand' would have 8 letters. We would say that such a progression is inadequate even though the progression is perfectly acceptable in pure mathematics and it exemplifies the natural number structure. If this is so, that means that in the application we ask for certain conditions that are not just structural, therefore not every progression is adequate, on the contrary, just one seems to be suitable.

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[23]Benacerraf P. 'What numbers could not be', page 291
[24]Benacerraf P. 'What numbers could not be', page 292
[25]Benacerraf P. 'What numbers could not be', page 292
[26]Frege G. *Grundlagen der Arithmetik*, § 56; English translation by J.L.Austin (1978)(Basil Blackwell, Oxford)
[27]Benacerraf P. 'What numbers could not be', page 290
[28]Benacerraf P. 'What numbers could not be', page 290

2) But, it seems that this progression doesn't even appear in Benacerraf's theory: there are no numbers 0,1, 2, 3, and so on, just numerals.

By identifying numbers with numerals we certainly don't have the problem of identifying numbers with any kind of objects, simply because 'there are no such things as numbers'[29]; all we have are numerals, words. It is unclear at this point in what way saying that there are not such things as numbers 'is not to say that there are not at least two prime numbers between 15 and 20.'[30] It seems that the property 'to be a prime number' is not related to numerals i.e. to the natural number structure, but to natural numbers. And not any sequence with a natural number structure is adequate: is we take for example Zermelo's ordinals than it's not clear which among Zermelo's ordinals are prime. What about, as Benacerraf says, any object that can play the role of 3; 'that is, any object can be the third element in some progression'[31]. Is this to mean that every object can be a prime number so that we can say that 6 is a prime number since in the progression 2, 4, 6, ... (which exemplifies the natural number structure therefore is perfectly acceptable) the number 6 plays the role of 3: number or numeral 3? Or even better, the numeral 6 plays the role of the numeral 3?

ii) *Ante rem* structuralism. As said in section 2, *ante re* structuralism is the doctrine according to which mathematics is concerned with abstract structures[32] and the elements of the structures have no properties beside the structural ones i.e. no non-structural properties. Mathematical objects (numbers, sets, ...) are just places within structures; e.g. real analysis is about the real number structure and everything we can say about real numbers consists in their 'structural' properties. It is not possible to postulate one real number because that would mean postulating one place within a structure which is not possible without invoking the structure as a whole. Mathematical entities have no internal properties and they are just positions in structures. It follows that they do not have identity outside the structure either;

> the various results of mathematics which seem to show that mathematical objects such as numbers do have internal structures, e.g., their identification with sets, are in fact interstructural relationships.[33]

According to Shapiro, even for a realist in ontology, questions like the Caesar problem need not to be answered, i.e., there is no answer:

> ...one can look into the identity between numbers denoted by different descriptions *in the language of arithmetic*. … But it makes no sense to pursue the identity between a place in the natural-number structure and some other object,… Identity between natural numbers is determinate; identity between numbers and other sorts of objects is not, and neither is identity between numbers and the positions of other structures.[34]

We have to ask question that are internal to the natural-number structure if we want to get determinate answers because mathematical objects are tied to the structure whose

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[29]Benacerraf P. 'What numbers could not be', page 294

[30]Benacerraf P. 'What numbers could not be', page 294

[31]Benacerraf P. 'What numbers could not be', page 291

[32]To say that a structure is abstract is to say that it can have more then one exemplification.

[33]'Mathematics as A Science of Patterns: Ontology and Reference', page 530

[34] *Philosophy of Mathematics - Structure and Ontology*, p. 79

places they occupy. So, even though, differently from Benacerraf, *ante rem* structuralists endorse the view according to which numbers are objects, they are objects of arithmetic. We can therefore ask questions about numbers if such questions are internal to the natural number structure, i.e. if they are about relations which can be defined in the language of arithmetic.

Adopting structuralism i.e.

> viewing mathematical objects as positions in patterns leads to a reconception of mathematical objects which defuses the objection to platonism based upon our inability to completely fix their identity.[35]

What are the difficulties in Shapiro's theory?

1) According to *ante rem* structuralism, the places in the natural-number structure can be occupied by places in other structures. This means that it is possible to have objects that are both places in the, e.g., S1 structure (there might be infinitely many objects) and exemplify the, e.g., S2 structure or occupy the places in such a structure. Such elements have therefore both certain properties which are internal in respect to the structure S1 (we shall call them S1-properties) and others which make them exemplify the structure S2 or are internal to the structure S2 (we shall call them S2-properties) and which don't correspond to the S1-properties. In that case the S1-properties are external, i.e. non structural in respect to the structure S2 and the S2-properties are external in respect to S1. If this is the case then such objects do have non structural properties in relations both to the structure S1 and S2, unless one structure is substructure of the other which is not a requested condition by Shapiro.

2) It might also be difficult to say in which way some properties of real numbers such as being transcendental can be treated as structural; this property appeals to the notion of polinomial which seems to be external to the structure.

3) It seems that questions as the Caesar problem are legitimate since places in the natural-number structure are also bona-fide objects of a system. If we ask "Is Julius Caesar=2" we identify Caesar with the natural number 2 - object of the natural number system so the question does have a determinate answer. If, on the other hand, those two objects do not belong to the same structure then the answer is obviously negative. Shapiro thinks that, since Caesar and the natural number 2 don't belong to the same structure, the Caesar problem does not need an answer but isn't the question "Do Caesar and the natural number 2 belong to the same structure?" just begging the question?

5. Structuralism and its solution of the epistemological problem

Structuralism allegedly solves the epistemological problem for Platonism, too. In fact,

> If we conceive of the numbers, say, as objects each one of which can be given to us in isolation from the others as we think of, say, chairs or automobiles, then it is difficult to avoid conceiving of knowledge of a number as dependent upon some sort of interaction between us and that number.[36]
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> ...

[35] 'Mathematics as A Science of Patterns: Ontology and Reference', page 530

[36] 'Mathematics as A Science of Patterns: Ontology and Reference', page 529

> I also think that viewing mathematics as a science of patterns promises to solve the Platonist's epistemological problems as well-or at least to make them less urgent-by showing that mathematical knowledge has a fairly central place in our general epistemological picture.

What is the structuralist's epistemology about? According to Shapiro, there are three ways of grasping a structure: abstraction or pattern recognition, linguistic abstraction and implicit definition.

One way of grasping a structure is through abstraction (or pattern recognition). We abstract a structure from one or more systems that have the same structure and grasp the common relations among the objects. This way is analogous to the way in which we grasp the type of a letter by observing different tokens of the letter and ignoring what is specific to a singular token like the colour, the height and the like. By abstraction we grasp small cardinal structures (the first few finite cardinal or ordinal structures) and it works the same as in the case of characters and strings: the child learns to recognize the 4 pattern after different groups of 4 objects have been pointed out to them. The next problem is how to grasp large cardinal structures (and then infinite systems and structures, too). Large cardinal structures are not apprehended by simple abstraction but children learn, during their linguistic development, to parse tokens of strings they have never seen and strings that may have no tokens at all:

> At some point, still early in our child's education, she develops an ability to understand cardinal and ordinal structures beyond those that she can recognize all at once via pattern recognition and beyond those that she has actually counted, or ever could count.[37]

In order to grasp the natural number structure we have to reflect on sequences of strokes that becomes longer and longer and form the notion of a never ending (in one direction) sequence of strokes:

> This is an infinite string, and so I cannot give a token of it in this book. The practice is to write something like this instead: | | | | . . . The point is that students eventually come to understand what is meant by the ellipses "'. . . "[38]

To obtain structures larger then the denumerable ones, we have to contemplate sets of rationals (as in Dedekind cuts) and in this way we contemplate the structure of the real numbers; we are talking in this case about linguistic abstraction. The third way to grasp a structure is through a direct description of it i.e. through its implicit definition, e.g., we can grasp the natural number structure by understanding the Peano's axioms which are its implicit definition. Shapiro defines the implicit definition in the following way:

> 'In the present context, an implicit definition is a *simultaneous* characterization of a number of items in terms of their relationships *to each other* . In contemporary philosophy, such definitions are sometimes calls 'functional definitions''[39]

[37]Shapiro *Philosophy of Mathematics - Structure and Ontology*, p. 117
[38]Shapiro *Philosophy of Mathematics - Structure and Ontology*, p. 119
[39]*Philosophy of Mathematics - Structure and Ontology*, p. 130

Both implicit definition and deduction support the view that mathematical knowledge is *a priori* :

> Thus, if sensory experience is not involved in the ability to understand an implicit definition, nor in the justification that an implicit definition is successful, nor in our grasp of logical consequence, then the knowledge about the defined structure(s) obtained by deduction from implicit definition is a priori.[40]

So, according to structuralists, structuralism resolves both

a) 'the plight of the mathematical Platonist arising from the existence of multiple reductions of the major mathematical theories'[41] and

b) the epistemological problem for Platonism due to the causally inert abstract mathematical entities)

What are the main problems for structuralist's epistemology?

1) How can we grasp a structure?

According to Shapiro, since structures are abstract, we do not have any causal contact with them. We do grasp small, finite structures by abstraction via pattern recognition.

> A subject views or hears one or more structured systems and comes to grasp the structure of those systems....The idea is that we grasp some structures through their systems just as we grasp character types through their tokens.[42]

This is how children grasp different types, e.g., letters: by looking at different tokens of letters showed them by their parents representing the same type. But, aren't types prior to tokens? Don't we have tokens in order to represent types, rather then the other way round? Children can learn about types through tokens because tokens have been already 'assigned' to types; the way we learn and the way we grasp are not necessarily the same.

2) What about infinite structures?

The way we grasp the natural-number structure is through its implicit definition, i.e. through a direct description of it. That means that we are supposed to grasp the natural-number structure via the understanding the Peano's axioms. Firstly, Shapiro doesn't say what it means to understand the Peano's axioms and what justify us in supposing that there is a structure described by the Peano's axioms. Secondly, it seems again a way of learning about the natural-number structure rather then a way of grasping it. Aren't the Peano's axioms the description of the natural-number structure we have somehow already grasped and we want to describe? It the Peano's axioms are a description of the natural-number structure, how can we describe a picture before grasping it in the first place? Doesn't that beg the question? Shapiro's describes the implicit definition as a 'common and powerful technique of modern mathematics:

> ...Typically, the theorist gives a collection of axioms and states that the theory is about any system of objects that satisfies the axioms. As I would put it, the axioms characterize a structure or a class of structure, if they characterize anything at all.[43]

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[40]Shapiro *Philosophy of Mathematics - Structure and Ontology*, p. 132

[41]Resnik M. D. (1982) 'Mathematics as a science of patterns: epistemology', NOUS 16, pp. 95-105, page 95

[42]Shapiro S. *Philosophy of Mathematics - Structure and Ontology*, p. 11

Here again, it is unclear how does the theorist get the (Peano's) axioms? Are they a result of the theorist's imagination or he grasps them in some way? If they are a result of the theorist imagination then it's unclear how we know that a structure corresponds to them; if the theorist has grasped them , the question is 'How?'. He couldn't have grasped them by grasping the structure since structures are abstract and causally inert (structuralism is supposed to solve the problematic Platonistic epistemology). He could have grasped them by grasping a system which exemplify the natural-number structure: one possible answer is by grasping the numerals which Shapiro denies;[44]the other possible answer is by grasping a spatio-temporal system which exemplify the natural-number structure. What about the real-number structure or other infinite structure? Since Shapiro is reluctant to assert the existence of an enough big number of physical objects in the universe when he criticizes eliminative structuralism he concludes:

> Because there are probably not enough *physical* objects to keep some theories from being vacuous, the eliminative structuralism must assume there is a large realm of abstract objects. Thus, eliminative structuralism looks a lot like traditional Platonism.[45]

According to Shapiro, one of the reasons why *ante rem* structuralism is the most acceptable version of structuralism is because it does not require a strong background ontology to fill the places of the various structures. But it seems that Shapiro might be after all committed to the existence of a 'large realm of abstract objects' as well as the eliminative version.

3) None of Shapiro's suggested way of grasping a structure explains how is it possible, if it is at all, to grasp a structure which no systems exemplify, it except for the structure itself. It seems that grasping such a structure would be as problematic as is grasping mathematical objects for Platonism whose epistemological problem structuralism allegedly solves, since structures are abstract and causally inert. The other versions of structuralism do not have to deal with such a case since there are no structures with no system that exemplifies them.

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[43]Shapiro S. *Philosophy of Mathematics - Structure and Ontology*, p. 12-3

[44]'I do not claim that the natural-number structure is somehow grasped by abstraction from numerals.' - *Philosophy of Mathematics - Structure and Ontology*, p. 137

[45]*Philosophy of Mathematics - Structure and Ontology*, p. 10

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