Low-Sensitivity, Low-Noise, Band-Rejection and All-Pass Active-RC Filters

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ABSTRACT—In this paper we introduce a new design procedure for low-sensitivity and low-noise filter biquads which have a symmetrical bridged-T passive-RC network. They are also low-power circuits, in that they use only one operational amplifier (opamp). The new design concept is based on the recently introduced “impedance tapering” method which was applied to allpole filters in [1], whereas in this paper it is extended to the design of filters with finite zeros. Thus, it is applied to two commonly used filter sections suitable for the realization of band-rejection and all-pass filters with low and medium pole-Q factors. In the new design procedure, the topology and component count remain the same; we just judiciously select the component values in order to reduce component tolerance sensitivity and improve noise performance. The sensitivity analysis is examined analytically and double-checked using PSpice Monte Carlo runs. In the PSpice noise analysis, a macro-model of the uA741 opamp is used. It is found that the minimum-sensitivity and minimum-noise filters coincide.

I. INTRODUCTION

In this paper we present low-sensitivity and low-noise design procedures for two commonly used 2nd-order active-RC single-opamp filter sections suitable for the realization of band-rejection (BR) and all-pass (AP) transfer functions (TF). Both are described in [2][3] and contain a physically symmetrical passive-RC sub-network (known as “bridged-T”). One is used for the realization of medium-pole-Q values (2 > q < 20) [3] pp. 60-61 for AP and pp. 62-64 for BR), while the other is used for low pole-Q values (q < 2) [3] pp. 48-49 for AP and pp. 50-51 for BR).

The new design method presented in this paper was first introduced in [1]. It is based on the appropriate impedance scaling of the filter components. It reduces the sensitivity to the passive components and improves the noise performance. The recently introduced design concept in [4], in which one half of a symmetrical passive-RC network is impedance scaled (by which the symmetrical-RC network becomes "potentially symmetrical"), is also investigated.

II. 2ND-ORDER BAND-REJECTION AND ALL-PASS BIQUADS

Consider two very common 2nd-order BR and AP filters; the "medium-Q" biquad in Fig. 1 and the "low-Q" in Fig. 2. A classification of single-opamp biquads was introduced in [2] and expanded in [5]. There, a 2nd-order active-RC filter as shown in Fig. 1 is called differential input (or type II), dual feedback (DF), class 4 (or "BP in the positive feedback loop"), which is designated by II-DF-4. Note that a ladder-RC network realizes the BP characteristics in the positive feedback loop, while in the signal forward path there is a "bridged-T" network. Recall that the bridged-T has the TF characteristic of the frequency-rejection network (FRN).

According to the classification in [5] the filter circuit in Fig. 2 is the II-SF-3 filter (type II or differential input, SF or single feedback, and class 3 or "FRN in the negative feedback loop"). It has a bridged-T in the negative feedback loop, and a ladder-RC network in the signal forward path. We can see that some kind of duality exists between filters in Fig. 1 and Fig. 2. The former combines a BP positive-feedback loop with constant negative feedback, while the latter has only an FRN-negative-feedback loop. A proper complementary transformation [6] applied to the circuit in Fig. 1 would have produced constant positive feedback

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TABLE I.  SECOND-ORDER FILTER COEFFICIENTS, POLE-ZERO FREQUENCY AND Q PARAMETERS IN TERMS OF COMPONENT VALUES.

<table>
<thead>
<tr>
<th>Filter circuit</th>
<th>(K)</th>
<th>(a_0 = \omega_0^2)</th>
<th>(a_1 = \omega_0 / q_p)</th>
<th>(\omega_0 = \omega_1)</th>
<th>(q_p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium-Q (Fig. 1)</td>
<td>1</td>
<td>(1 / R_C (C_1 + C_2) + R_C (1 - \alpha \beta) / R_C (C_1 + C_2))</td>
<td>(1 / \sqrt{R_C (C_1 + C_2) / (R_C (C_1 + C_2) + R_C (1 - \alpha \beta))})</td>
<td>1</td>
<td>(\sqrt{R_C (C_1 + C_2) / (R_C (C_1 + C_2) + R_C (1 - \alpha \beta))})</td>
</tr>
<tr>
<td>Low-Q (Fig. 2)</td>
<td>(\alpha = -R_C / (R_C + R_i))</td>
<td>(1 / R_C (C_1 + C_2))</td>
<td>(C_1 / R_C C_2)</td>
<td>1</td>
<td>(\sqrt{R_C (C_1 + C_2) / (R_C (C_1 + C_2) + R_C (1 - \alpha \beta))})</td>
</tr>
</tbody>
</table>

Therefore, they are presented in the first column of Table 2, where \(x\) represents each of the passive components.

By "non-standard" design we mean the "impedance tapering" design procedure of low-sensitivity allpole active-RC filters, which was first introduced in [1]. In the filter examples in this paper we perform impedance tapering using (3), and we obtain the sensitivities in the second column of Table 2.

For class-4 circuits, the method of minimizing the sensitivity of the pole \(Q\), \(q_p\), with respect to the positive feedback resistors, \(R_1\) and \(R_2\), is given by:

\[ q_p = \left(1 - \beta \cdot q_p / \omega_0 \cdot \omega_0^2\right)^{-1}. \]  

(9)

The sensitivity of the pole \(Q\), \(q_p\), (or \(a_1\)), to the gain \(\beta\) readily follows from (9), and it is given by:

\[ S_{\beta q_p} = -S_{q_p} = q_p / \hat{q} - 1. \]  

(10)

According to (10) the sensitivity is reduced as the value of the passive pole \(Q\), \(q_p\), increases [e.g. by increasing impedances \(R_1\) and \(C_1\) as in (3)]. Will the decrease in sensitivity in (10) really reduce the sensitivity to the tolerances of the two passive-feedback resistors \(R_1\) and \(R_2\)? The gain \(\beta\) sensitivity to the resistors \(R_1\) and \(R_2\) readily follows from (2), and it is given by:

\[ S_{\beta R_1} = -S_{R_1} = 1 - 1 / \beta. \]  

(11)

Finally, the relative variation of pole \(Q\), \(q_p\), due to variations in resistors \(R_1\) and \(R_2\), is given by:

\[ S_{R_1 q_p} = -S_{q_p} = S_{R_1} + S_{R_2} = (q_p / \hat{q} - 1) \cdot (1 - 1 / \beta). \]  

(12)

If we express the gain \(\beta\) from (9) and substitute it into (12) we have the following form of the sensitivity:

\[ S_{R_1 q_p} = -S_{q_p} = q_p (1 / \hat{q} - \omega_0 / \omega_0^2) - 1. \]  

(13)

Introducing (3) into (13) we obtain the sensitivity in the form:

\[ S_{R_1 q_p} = -S_{q_p} = q_p (1 / \hat{q} - \omega_0 / \omega_0^2) - 1. \]  

(14)

Note that the sensitivity in (13) [or (14)] and all sensitivities in Table 1 are proportional to the pole \(Q\), \(q_p\). This is the characteristic of "medium-Q" filters [2]. This means that one does well to select the filter type yielding the lowest pole \(Q\), for a given specification.

A glance at the sensitivity in (14), and those in the second column of Table 2, shows that some of them are partially proportional\(^1\) to the capacitive scaling factor \(c\), but exclusively inversely proportional to the resistive scaling factor \(r\). Other expressions, which include terms \((rc)\), will be small since they

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\(^1\) By "partial proportionality" we mean that \(c\) will appear partially in the numerator, partially in the denominator of the sensitivity expressions.
are multiplied by the small quantities $\alpha(1-\beta)$, $(1-\alpha)$ and $(1-\alpha\beta)$ (note that the gain $\beta$ will generally be between unity and, say, 2.5 and $\alpha$ less than unity). Thus, resistive impedance scaling with equal capacitors ($c=1$ and $r=1$) reduces the coefficient sensitivities to all passive filter components. To check the above conclusions regarding sensitivity, we design filters with various resistance ($r$) and capacitance ($c$) ratios in the following example.

**Example:** Let's realize a BR filter having 1kHz center frequency and pass-band range of 200Hz. To obtain this selectivity we need a pole-$Q$ factor of $q_p=0.005$, $B=5$. The magnitude of the TF characteristic is shown in Fig. 3.

![Figure 3. The 2nd-order filter BR TF magnitude ($q_p=1kHz$, $q_c=5$).](image)

When we build filters using integrated circuit technology, we must calculate resistors and capacitors in such a way that we do not exceed the upper limit of total capacitance $C_{TOT}=C_1+C_2$. With the total capacitance $C_{TOT}$ given, we choose the capacitance ratio $c$ [according to (3)] and calculate $C$ using

$$C=C_{TOT}\, (1+1/c)^{-1}. \tag{15}$$

For the example of total capacitance value $C_{TOT}=300pF$, which is realizable on the chip, we obtain the component values of the filters in Table 3. In addition, in the last two columns of Table 3, pole- and zero-$Q$ factors of the passive bridged-T sub-network are given. Corresponding Monte Carlo (MC) runs with 1% Gaussian distribution, zero-mean resistors and capacitors were carried out using PSpice and presented in Fig. 4.

**TABLE III. COMPONENT VALUES OF 2nd-ORDER FILTERS WITH VARIOUS SCALING FACTORS (RESISTORS IN [KΩ], CAPACITORS IN [PF]).**

<table>
<thead>
<tr>
<th>No.</th>
<th>$r$</th>
<th>$c$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\hat{q}$</th>
<th>$q_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1061</td>
<td>150</td>
<td>1061</td>
<td>150</td>
<td>0.933</td>
<td>3.0</td>
<td>0.333</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>2652</td>
<td>60</td>
<td>663</td>
<td>240</td>
<td>0.962</td>
<td>1.312</td>
<td>0.190</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4</td>
<td>1326</td>
<td>60</td>
<td>1326</td>
<td>240</td>
<td>0.956</td>
<td>2.25</td>
<td>0.222</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>2122</td>
<td>150</td>
<td>530</td>
<td>150</td>
<td>0.933</td>
<td>1.50</td>
<td>0.333</td>
<td>1.0</td>
</tr>
</tbody>
</table>

![Figure 4. MC runs of imp.-tapered 2nd-order BR filters given in Table 3.](image)

We can see that filter no. 4 with equal capacitors ($c=1$) and resistor ratio $r=1$ has minimum sensitivity. In [3] are given design procedures for min.-GSP biquads (the GSP gives a measure of a filter’s magnitude sensitivity to the open-loop opamp gain ($\hat{A}$) variation). The optimum trade-off in the design of the filter circuit in Fig. 1 is to choose resistor ratio $r=1$ thus reducing passive sensitivity, and to calculate capacitor ratio $c$ for min. GSP (from [3] p. 63), which provides a circuit with reduced active sensitivity, as well.

Note that for the BR filter case the coefficient $b_1$ in (1) does not exist. For the AP filter case, $b_1$ exists, and the sensitivities of the coefficient $b_1$ to the components are calculated. Those sensitivities have very similar form to the $a_i$ sensitivities in Table 2, and therefore, will not be presented. Additionally, the sensitivity of the AP filter has been analysed by MC runs, and it can be concluded that the same design strategies applied in the BR filter design can efficiently be extended to the AP filter design.

### B. Noise Analysis of a Band-Rejection Filter

We demonstrate that band-rejection active-RC filters that are designed for minimum sensitivity to component tolerances are also superior in terms of low output and input thermal noise. Noise effects are calculated with the simulation program PSpice with a macro-model of operational amplifier uA741 and with the circuit elements at their nominal values.

Figures-of-merit such as dynamic range and noise factor will not be calculated; instead the curves that represent the output and input noise spectral densities will be observed and compared. We recall that with lower output noise level we obtain higher dynamic range, and with lower input noise level we obtain lower noise factor.

To analyze noise contributions we use the same examples, given in Table 3. The corresponding output and input noise spectral densities are shown in Fig. 5.

![Figure 5. Output and input noise spectral density of impedance-tapered 2nd-order BR filters given in Table 3.](image)

Observing the noise spectral density curves in Fig. 5, we conclude that the filter with the lowest noise is filter no. 4, which has minimum sensitivity as well.

Sensitivity performance are dependent only on the values of the component ratios and the gain $\beta$, while noise is dependent on the resistor values in the circuit and the operational amplifier itself. It is luck that the min. sensitivity design procedure of the BR active-RC filter in Fig. 1 provides min. noise performance, as well.

### C. Sensitivity Reduction using Potential Symmetry

In the design concept introduced recently in [4], instead of general scaling factors $r$ and $c$ [as in (3)] we apply impedance scaling to the symmetrical passive-RC network; it becomes "potentially symmetrical". Therefore we use scaling factor $\rho$:

$$R_1=(1+\rho)R; \quad C_1=C; \quad R_2=\rho(1+\rho)R; \quad C_2=C/\rho. \tag{16}$$

Note that by increasing the impedance-scaling factor $\rho$ of one half of a symmetrical passive RC-network, we increase its pole-$Q$ factor, $\hat{q}$, towards the upper limit of (never accessible) 0.5 (see [2]), while the zero-$Q$ factor, $q_c$, of the bridged-T is always equal to unity regardless of the $\rho$ value.

With (16) we obtain the sensitivity relations given in the third column of Table 2. Furthermore, introducing (16) into (13) we obtain the sensitivity of the positive feedback gain $\beta$ to the resistors $R_1$ and $R_2$ in the form:

$$S_{\beta R_1} = -S_{\beta R_2} = q_c - 1. \tag{17}$$

A glance at the sensitivity in (17), and those in the third column of Table 2, shows that increasing the scaling factor $\rho$ reduces the sensitivities to the passive component tolerances very slightly, and to the feedback resistors $R_1$ and $R_2$ not at all. In contrast, in the case of "Split-Feedback FRN" in [4], increasing the $\rho$ reduces the sensitivity. From this we conclude that the potential symmetry can effectively be used only in the design of few filter circuits. To double-check the above conclusions we designed the filter in Fig. 1, with two values of $\rho$. The component values of the resulting filters are in Table 4, MC runs are in Fig. 6, and noise analysis is in Fig. 7.

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In what follows, we design the filter circuit of Fig. 2. In the first step we realize the desired value of pole $Q$, $Q_p$. We choose a capacitor ratio $c$, and then calculate a resistor ratio $r$ using

$$r = q_p^2 \cdot 2 + c^2 + c.$$  \hspace{1cm} (18)

Next, to realize the desired pole and zero frequencies $\omega_0$ and $\omega_z$, we calculate the passive elements in the bridged-T network using (4) above. Finally, to realize the desired zero-Q factor, $q_z$ (see $q_z$ in Table 1), we have to find the resistors $R_1/R_2=1/\alpha - 1/\beta - 1$. In the design procedure, we distinguish between the two main cases:

(i) to design a band-rejection (BR) filter, we calculate $\beta$ from (7) and then $R_1/R_2 - \beta - 1$.

(ii) to design an all-pass (AP) filter, we calculate $\beta$ from (8) and then $R_1/R_2 - \beta - 1$.

Example: Let’s realize a BR filter having 1kHz center frequency and pass-band range of 500Hz, i.e. the pole $Q_p = 500$ and $\beta = 2$. The magnitude of the TF characteristic is shown in Fig. 8a. Filter component values with three values of $c$ are presented in Table 5, and corresponding MC runs are in Fig. 8b-d. The output and input noise spectral densities are shown in Fig. 9.

Observing the MC runs in Fig. 8, we conclude that filter no. 2 with equal capacitors ($c=1$) has slightly (almost negligibly) lower sensitivity than the other two filters, 1 and 3. This very small sensitivity reduction, obtained by varying impedance scaling factors $c$ and $r$, is characteristic of the "low-Q" circuits (see [7]). In addition, filter no. 2 has minimum component spread (resistor ratio). Recall that the function $r(c) = c^2 + c$ reaches its min. value of 2 when the value of $c=1$. In that case, (18) reaches its minimum and simplifies into $r = 4q_p^2$. This is another reason for choosing equal capacitors. Note the characteristic proportionality of the component spread to the squared pole $Q$, $q_p$, by "low-Q" circuits [7]. And finally, observing the curves in Fig. 9, we conclude that the filter with the lowest noise is again filter no. 2.

III. LOW-Q 2ND-ORDER BR AND AP BIQUAD

In what follows we design the filter circuit of Fig. 2. In the first step we realize the desired value of pole $Q$, $q_p$. We choose a capacitor ratio $c$, and then calculate a resistor ratio $r$ using

$$r = q_p^2 \cdot 2 + c^2 + c.$$  \hspace{1cm} (18)

Next, to realize the desired pole and zero frequencies $\omega_0$ and $\omega_z$, we calculate the passive elements in the bridged-T network using (4) above. Finally, to realize the desired zero-Q factor, $q_z$ (see $q_z$ in Table 1), we have to find the resistors $R_1/R_2 = 1/\alpha - 1/\beta - 1$. In the design procedure, we distinguish between the two main cases:

(i) to design a band-rejection (BR) filter, we calculate $\beta$ from (7) and then $R_1/R_2 - \beta - 1$.

(ii) to design an all-pass (AP) filter, we calculate $\beta$ from (8) and then $R_1/R_2 - \beta - 1$.

Example: Let’s realize a BR filter having 1kHz center frequency and pass-band range of 500Hz, i.e. the pole $Q_p = 500$ and $\beta = 2$. The magnitude of the TF characteristic is shown in Fig. 8a. Filter component values with three values of $c$ are presented in Table 5, and corresponding MC runs are in Fig. 8b-d. The output and input noise spectral densities are shown in Fig. 9.

Observing the MC runs in Fig. 8, we conclude that filter no. 2 with equal capacitors ($c=1$) has slightly (almost negligibly) lower sensitivity than the other two filters, 1 and 3. This very small sensitivity reduction, obtained by varying impedance scaling factors $c$ and $r$, is characteristic of the "low-Q" circuits (see [7]). In addition, filter no. 2 has minimum component spread (resistor ratio). Recall that the function $r(c) = c^2 + c^2$ reaches its min. value of 2 when the value of $c=1$. In that case, (18) reaches its minimum and simplifies into $r = 4q_p^2$. This is another reason for choosing equal capacitors. Note the characteristic proportionality of the component spread to the squared pole $Q$, $q_p$, by "low-Q" circuits [7]. And finally, observing the curves in Fig. 9, we conclude that the filter with the lowest noise is again filter no. 2.

Table IV. Component Values of 2nd-Order BR Filters with Potentially Symmetrical Bridged-T.

<table>
<thead>
<tr>
<th>No.</th>
<th>$\rho$</th>
<th>$R_1$</th>
<th>$C_1$</th>
<th>$R_2$</th>
<th>$C_2$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$q_z$</th>
<th>$q_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>1</td>
<td>21222</td>
<td>150</td>
<td>530</td>
<td>150</td>
<td>0.933</td>
<td>1.50</td>
<td>0.333</td>
<td>1.0</td>
</tr>
<tr>
<td>2)</td>
<td>4</td>
<td>3316</td>
<td>240</td>
<td>530</td>
<td>60</td>
<td>0.911</td>
<td>1.80</td>
<td>0.444</td>
<td>1.0</td>
</tr>
</tbody>
</table>

In what follows we design the filter circuit of Fig. 2. In the first step we realize the desired value of pole $Q$, $q_p$. We choose a capacitor ratio $c$, and then calculate a resistor ratio $r$ using

$$r = q_p^2 \cdot 2 + c^2 + c.$$  \hspace{1cm} (18)

Next, to realize the desired pole and zero frequencies $\omega_0$ and $\omega_z$, we calculate the passive elements in the bridged-T network using (4) above. Finally, to realize the desired zero-Q factor, $q_z$ (see $q_z$ in Table 1), we have to find the resistors $R_1/R_2 = 1/\alpha - 1/\beta - 1$. In the design procedure, we distinguish between the two main cases:

(i) to design a band-rejection (BR) filter, we calculate $\beta$ from (7) and then $R_1/R_2 - \beta - 1$.

(ii) to design an all-pass (AP) filter, we calculate $\beta$ from (8) and then $R_1/R_2 - \beta - 1$.

Example: Let’s realize a BR filter having 1kHz center frequency and pass-band range of 500Hz, i.e. the pole $Q_p = 500$ and $\beta = 2$. The magnitude of the TF characteristic is shown in Fig. 8a. Filter component values with three values of $c$ are presented in Table 5, and corresponding MC runs are in Fig. 8b-d. The output and input noise spectral densities are shown in Fig. 9.

Observing the MC runs in Fig. 8, we conclude that filter no. 2 with equal capacitors ($c=1$) has slightly (almost negligibly) lower sensitivity than the other two filters, 1 and 3. This very small sensitivity reduction, obtained by varying impedance scaling factors $c$ and $r$, is characteristic of the "low-Q" circuits (see [7]). In addition, filter no. 2 has minimum component spread (resistor ratio). Recall that the function $r(c) = c^2 + c^2$ reaches its min. value of 2 when the value of $c=1$. In that case, (18) reaches its minimum and simplifies into $r = 4q_p^2$. This is another reason for choosing equal capacitors. Note the characteristic proportionality of the component spread to the squared pole $Q$, $q_p$, by "low-Q" circuits [7]. And finally, observing the curves in Fig. 9, we conclude that the filter with the lowest noise is again filter no. 2.

Table V. Component Values of 2nd-Order BR Filters with Various Scaling Factors.

<table>
<thead>
<tr>
<th>No.</th>
<th>$r$</th>
<th>$c$</th>
<th>$R_1$</th>
<th>$C_1$</th>
<th>$R_2$</th>
<th>$C_2$</th>
<th>$\alpha$</th>
<th>$q_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>1/4</td>
<td>6631</td>
<td>240</td>
<td>265</td>
<td>60</td>
<td>0.833</td>
<td>0.333</td>
<td>1.0</td>
</tr>
<tr>
<td>2)</td>
<td>1</td>
<td>6631</td>
<td>240</td>
<td>265</td>
<td>60</td>
<td>0.888</td>
<td>0.222</td>
<td>1.0</td>
</tr>
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<td>3)</td>
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<td>240</td>
<td>265</td>
<td>60</td>
<td>0.952</td>
<td>0.095</td>
<td>1.0</td>
</tr>
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REFERENCES


