Image Sharpening Using Image Sequence and Independent Component Analysis

Ivica Kopriva¹, Qian Du² and Harold Szu¹

¹Digital Media RF Lab, Department of Electrical and Computer Engineering
George Washington University, Washington DC 20052, USA
²Department of Electrical Engineering and Computer Science
Texas A&M University-Kingsville, Kingsville, Texas 78363
e-mail: ikopriva@gwu.edu

ABSTRACT

The novel approach to the image sharpening problem is proposed in this paper. It is based on the application of the independent component analysis (ICA) algorithm on the image sequence with the appropriate time displacement between the image frames. The novelty is in the data representation required by the ICA algorithms where each selected image frame has been used as a sensor implying that underlying sources are temporally independent. The proposed concept enables blurring effects contributed by atmospheric turbulence to be extracted as separate physical sources. It has been ensured through images registration technique that motion of the video recorder is compensated. Encouraging preliminary results were obtained when ICA algorithm has been applied on the experimental data (video sequence) with the known ground truth. It has been verified that extracted spatial turbulence patterns are highly impulsive with Gaussian exponent between 0.5 and 0.6 where Laplacian distribution is characterized with Gaussian exponent 1.

Key words: image sharpening, image sequence, independent component analysis.

1.0 INTRODUCTION

It is known that performance of the imaging systems is limited by diffraction¹⁴ due to the fact that point sources at the object plane are spread in the far field image plane which sets the Rayleigh resolution criterion: the finest structure that a system can resolve is given by

\[ \delta_{X, res} = 1.22 \frac{\Delta f_y}{\lambda} = 1.22 \frac{\Delta f}{B} \]  

(1)
where $\lambda$ is the wavelength, $f$ is the focal length of the lens, and $B$ is the system aperture size. Resolution beyond the classical diffraction limit may be possible in certain cases\textsuperscript{2, 5-8, 20}. However, it was pointed out in Ref.\textsuperscript{17} and \textsuperscript{18} that atmospheric turbulence caused by both space and time random fluctuations of the refractive index, that itself is caused by microscale temperature fluctuations, will limit performance of the imaging system much beyond the classical Raleigh’s diffraction limit. In this paper we propose a novel approach to image sharpening based on the processing of the image sequence by means of the independent component analysis algorithm (ICA)\textsuperscript{9} treating each video sequence frame as a sensor. The physical justification of the adopted data model has not been derived yet and will be reported elsewhere. We describe our approach in section 2 where also brief comparison with the multispectral/hyperspectral imaging is given. Brief description of the fourth order (FO) statistics based ICA algorithm for Joint Approximate Diagonalization of Eigenmatrices (JADE) is also given in section 2. Preliminary experimental results are given in section 3 while conclusion is given in section 4.

2.0 PROBLEM FORMULATION

Inspired by concept presented in Ref. 13 we have formulated a novel way how to represent image sequence in the ICA framework that in a way has full analogy with how the ICA is used in the analysis of multispectral and/or hyperspectral data\textsuperscript{11, 14, 15}. The ICA methods are applied on the multispectral /hyperspectral image cube presuming that hidden sources have different spectral signatures. As illustrated on the left picture of Figure 1 each spectral component is treated as a sensor in the ICA framework. By analogy with multispectral/hyperspectral imaging frequency dimension ($\omega$) can be replaced by time dimension ($t$) in which case optical imaging based on the image sequence is obtained, right picture on Figure 1. The same concept of the image cube can be used to represent image sequence in the ICA framework where each video frame is treated as a sensor. It assumes that an image sequence is acquired with a single camera and that motion effects can be neglected. The important novelty of this data representation scheme is that it uses the temporal information contained in an image sequence. Intensity variation of the objects through time is what can make the measurements linearly independent. The underlying assumption is that different sources will have different (statistically independent) time realizations. This represents basis for image sharpening through extraction of the turbulence contributions as separate sources expected to be highly super-Gaussian. We use the standard linear scalar data model with additive noise, Eq.(1).

$$ u = As + \nu $$  \hspace{1cm} (1)

Individual component of the data vector $u$ is represented as $u(t_k, x, y)$ where $t_k$ is video frame index and $(x, y)$ are pixel coordinates on the related video frame and $A$ and $s$ represent unknown mixing matrix and source.
signal vector respectively while \( v \) is spatially independent additive noise assumed to be Gaussian. On the component level Eq.(1) can be written as

\[
\mathbf{u}_k(t_k, x, y) = \sum_{n=1}^{N} a_{kn} (\Delta t_{kn}) s_n(t_1, x, y)
\]  

(2)

Eq.(2) means that each measured data at spatial coordinate \((x,y)\) is a superposition of different source signals existed at the same spatial coordinate \((x,y)\) at some reference time point \(t_1\). Relative contributions between some time point \(t_k\) and reference time point \(t_1\) are contained in the unknown mixing matrix coefficients \(a_{kn}\). One of the source components correspond with our object of interest while the rest are different spatial realizations of the turbulence. The idea that relative change of the intensity between time point \(t_k\) and reference time point \(t_1\) can be expressed on the adopted way (2) is in agreement with the Taylor’s frozen hypothesis\(^\text{18}\) which states that change of the refractive index over short time interval \(t_2 > t_1\) remains fixed except for translation with uniform transfer velocity:

\[
n_1(x, y, t_2) = n(x - v_x(t_2 - t_1), y - v_y(t_2 - t_1), t_1)
\]  

(3)

Figure 1. Representation of multispectral /hyperspectral data for ICA analysis using the concept of image cube (left). Using the same concept image sequence can be represented in the ICA framework where each frame is treated as a sensor (right).

The strategy of the ICA algorithms is to find linear transform \(W\)

\[
\mathbf{z} = \mathbf{Wu} = \mathbf{WA}s + \mathbf{Wv} = \mathbf{Q}s + \mathbf{Wv}
\]  

(4)

such that components of the vector \(\mathbf{z}\) be as much statistically independent as possible. Based on the assumption that source signals \(s\) are mutually statistically independent and non-Gaussian (except maybe one that is allowed to be Gaussian) the vector \(\mathbf{z}\) will represent source vector signals \(s\) up to the permutation and
scale factor. Here we have used the FO cumulant based ICA algorithm JADE\textsuperscript{8} where statistical independence is achieved through minimisation of the squares of the FO cross-cumulants between the components $z_i$
\begin{equation}
W = \arg \min \sum_{i\neq j} \left| \hat{C}_4(z_i, z_j, z_k, z_l) \right|^2
\end{equation}
where $\hat{C}_4(z_i, z_j, z_k, z_l)$ are sample estimates of the related fourth-order cross-cumulants i.e.\textsuperscript{9,10}
\begin{equation}
\hat{C}_4(z_i, z_j, z_k, z_l) = \left\{ z_i z_j z_k z_l \right\} - \left\{ z_i z_j \right\} \left\{ z_k z_l \right\} - \left\{ z_i z_k \right\} \left\{ z_j z_l \right\} - \left\{ z_i z_l \right\} \left\{ z_j z_k \right\}
\end{equation}
where $\left\{ \right\}$ denotes expectation operator. In Eq.(5) the sum is over all the quadruples $(i,j,k,l)$ of indices with $i \neq j$ so that for every $i, j (i \neq j)$ we have square matrix defined by $[k,l]$ pairs where $(k,l = 1,\ldots,N)$. The additional advantage of using FO cumulants based ICA algorithm is its capability to suppress additive Gaussian noise based on the known property that FO cumulants are blind w.r.t. Gaussian noise\textsuperscript{9}. Based on the multilinearity property of the cumulants\textsuperscript{9,10} we can write any of the FO cross-cumulants as
\begin{equation}
C_{kl}(z_i, z_j) = \sum_{n=1}^{N} q^n_k q^n_j C_4(s_n) + \sum_{n=1}^{N} w^n_k w^n_j C_4(v_n)
\end{equation}
Now thanks to the fact that FO cumulants are blind w.r.t. Gaussian noise i.e. $C_4(v_n) = 0$ Eq.(7) becomes
\begin{equation}
C_{kl}(z_i, z_j) = \sum_{n=1}^{N} q^n_k q^n_j C_4(s_n) \quad k + l = 4 \text{ and } k, l \in \{1,2,3\}
\end{equation}
So except recovering the unknown source signals the FO cumulants based ICA methods have additional advantage to suppress additive Gaussian noise. Another critical factor in the ICA applications is the level of (non)singularity of the mixing matrix $A$. Although it is unknown we must assumed it to be invertible. In the context of our application it is affected by selection of time displacement $\Delta t$ between the video frames used as sensors in the ICA data model (1). If we assume that image sequence is short enough such that motion effects can be neglected then we must ensure that video frames taken into ICA data model must be different enough in order to make measurements linearly independent. It seemed natural to use Kullback-Leibler divergence to measure the mutual information between the images $p$ and $q$\textsuperscript{11,12}
\begin{equation}
D(p;q) = L(p;q) + L(q;p)
\end{equation}
\begin{equation}
L(p;q) = \sum_{k=1}^{K} p_k \log \frac{p_k}{q_k} \quad L(q;p) = \sum_{k=1}^{K} q_k \log \frac{q_k}{p_k}
\end{equation}
Because Kullback distance is not symmetrical both distances $L(p;q)$ and $L(q;p)$ are used in (9). When two images are the same the Kullback distance $D(p,q)$ will be zero. In order to ensure linear independence between selected video frames time displacement $\Delta t$ has to be chosen such that average Kullback distance $\overline{D(p,q)}$ is maximized i.e.
\[ \Delta t = \arg \max \frac{2(N-2)!}{N!} \sum_{k=1}^{N} D(p_k, q_{k+1}) \] (11)

3.0 EXPERIMENTAL RESULTS

In order to verify proposed image sharpening technique we have used an image sequence of the Washington monument consisted of 100 frames. In selecting number of frames we have taken care not to introduce motion effects. Figure 2 shows five frames in the top row with displacement between the frames \( \Delta t = 24 \). Figure 3 shows average Kullback distance between the five sensor frames for different time displacements. The largest value that corresponds with minimal information redundancy between the frames is obtained for \( \Delta t = 24 \) frames displacement. Bottom row shows extracted source images using JADE ICA algorithm\(^8\). In accordance with intuitive justification of the adopted ICA data model (1)/(2) one sharpened image of the original object and several source images that correspond with different turbulence realizations should be extracted. We have used Cany’s edge extraction algorithm to quantify the results of the proposed image sharpening algorithm. Edges extracted from data images (top row) and source images (bottom row) using Cany’s method for edge extraction are shown on Figure 4. Note the extracted source image that corresponds with the original object, Washington monument, (third in the bottom row) gave the best edge extraction result when compared to all data images (top row). Edges that correspond with the windows that exist on the top of the Washington monument are extracted from the third source image as well as edges that correspond with the background. Note also that on original data images edges that correspond with the windows are partially extracted only on the third data image. Note that edges that correspond with the shape of the Washington monument are completely continuous and relatively smooth on the third source image what is not the case with the edges extracted from all data images. We also want to mention that experiments with less than five data frames gave worse results than presented case although greater displacement between the frames was possible to select. That suggests that certain turbulence contribution remained on the extracted source image as an un-modeled source.
Based on the highly stochastic nature of the turbulence\textsuperscript{17,18} it is reasonable to expect that extracted spatial turbulence patterns will have highly super-Gaussian distributions. It is possible to characterize the stochastic process through the parameter called kurtosis:

\begin{equation}
\kappa(z_i) = \frac{C_4(z_i)}{C_2(z_i)} = \frac{E[z_i^4]}{E[z_i^2]} - 3
\end{equation}

that is positive for super-Gaussian processes. Table 1 presents results of the estimated values of the kurtosis parameter for both data and extracted source images. As expected extracted source images that correspond with turbulence are highly super-Gaussian with the kurtosis parameter greater than 16. If we use generalized Gaussian distribution\textsuperscript{16}

\begin{equation}
p(y) = \frac{r}{2\sigma^r(1/r)} \exp \left( -\frac{1}{r} \frac{|y|^{r}}{\sigma^{r}} \right)
\end{equation}
the kurtosis as a function of the Gaussian exponent $r$ is given with:

$$k_4(r) = \frac{\Gamma\left(\frac{5}{r}\right)\Gamma\left(\frac{1}{r}\right)}{\Gamma^2\left(\frac{3}{r}\right)} - 3$$

Figure 5 shows kurtosis as a function of the Gaussian exponent for super-Gaussian processes. Based on the results given in Table 1 it can be seen that kurtosis of the extracted source signals that correspond with turbulence patterns gives Gaussian exponent between 0.5 and 0.6. For the sake of comparison we comment here that Laplacian distribution has Gaussian exponent equal to 1. This result confirms our expectations about highly impulsive nature of the turbulence.

Figure 3. Average Kulback distance between data images for different displacements between the sensor frames.
Figure 4. Edges extracted from data images (top row) and source images (bottom row) using Cany’s method for edge extraction. Note the extracted source image that corresponds with the original object, Washington monument, (third in the bottom row) gave the best edge extraction result when compared to all data images (top row).

Table 1. Kurtosis of data and source images.

<table>
<thead>
<tr>
<th>Data/Source index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data kurtosis</td>
<td>0.0748</td>
<td>0.0917</td>
<td>0.1264</td>
<td>0.0899</td>
<td>0.2699</td>
</tr>
<tr>
<td>Source kurtosis</td>
<td>22.274</td>
<td>16.9</td>
<td>0.086</td>
<td>17.17</td>
<td>25.504</td>
</tr>
</tbody>
</table>
4.0 CONCLUSION

The novel approach to the image sharpening problem is proposed in this paper. It is based on the application of the independent component analysis algorithm on the image sequence with the appropriate time displacement between the image frames where selected image frames have been used as sensors implying that underlying sources are temporally independent. We have demonstrated the capability of the proposed concept to extract blurring effects contributed by atmospheric turbulence as separate physical sources. When Cany’s method for edge extraction has been used to quantify image sharpening results the best results were obtained with the extracted source image that corresponds with the original object. We have also shown that extracted spatial turbulence patterns are highly impulsive with the kurtosis parameter greater than 16 and Gaussian exponent between 0.5 and 0.6 where Laplacian distribution is characterized with the Gaussian exponent 1.

REFERENCES

2. Ref. 1, Frequency Analysis of Optical Imaging Systems, Chap. 6, pp.126-165.


