PERFORMANCE EVALUATION OF ROOT BASED DIRECTION FINDING ALGORITHMS USING HARDWARE EMULATION OF THE ANTENNA ARRAY

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Abstract: To validate the theoretically predicted performance of root based direction finding (DF) algorithms an experimental test bed was assembled covering the 1850-1910 MHz band. A four-element antenna array with uniform interelement spacing was emulated using variable phase shifters creating what is essentially a decoupled array environment. The array outputs were down converted using standard analogue circuits and fed to a four-channel receiver providing four separate digitized data channels. The data were processed with the Modified Root Pisarenko (MRP) algorithm. The results show that using an equivalent physical array aperture of only 1.5 wavelength the directions of arrival (DOA) in a decoupled array environment can be estimated with errors smaller than 0.01°.

INTRODUCTION
Super-resolution DF algorithms fall into two basic categories: search based algorithms such as MUSIC [2], and the root based algorithms such as Root-MUSIC [3] or the MRP [1]. In addition to being computationally very demanding search algorithms require complex array calibration procedures. On the other hand, while root-based algorithms afford computational efficiency and require no array calibration, they assume that the elements of the array steering vector have the complex exponential form of the classical array factor of a linear array of uniformly spaced elements. Unfortunately this idealized form of the steering vector can not be realized in a physical antenna array due to the mutual coupling which can cause significant angle estimation errors when the root-based DF algorithms are applied without some form of compensation [1] [4]. Effective compensation for the effects of mutual coupling can be realized either through the use of a decoupling transform [1] or by employing extra “dummy” elements to equalize the active element radiation patterns [6]. The DOA estimation performance achievable with root algorithms in an ideal decoupled array environment may therefore be regarded as a benchmark that one can hope to reach through the application of compensation techniques of increasing sophistication. Since a physical antenna array can not provide an ideal decoupled environment the experimental approach adopted herein was to emulate the ideal array steering vector using variable RF phase shifters thus creating experimentally what is essentially a decoupled array environment. The array outputs were down converted using standard analogue circuits and fed to a four-channel receiver providing four separate digitized data channels. The data were processed with the MRP algorithm. The results show that using an equivalent physical array aperture of only 1.5 wavelength the DOA in a decoupled array environment can be estimated with errors smaller than 0.01°.

BRIEF SUMMARY OF THE MRP ALGORITHM
The problem of estimating the directions of arrival of $L$ uncorrelated plane waves incident on a linear array of $N$ uniformly spaced decoupled elements is described by

$$z_n(t) = \sum_{\ell} \exp \left( j \frac{2\pi}{\lambda} nd \cos \psi_\ell \right) s_\ell(t) + \nu_n(t)$$  \hspace{1cm} (1)$$

where $z_n(t)$ is the output of the $n$-th receiver, $\psi_\ell$ is angle between the array axis and the direction of incidence of the $\ell$-th signal $s_\ell(t)$ and $\nu_n(t)$ represents the noise contribution from the $n$-th receiver. The MRP algorithm computes the incidence angles $\psi_\ell$ from the roots of the $2L$ degree polynomial $P_{2L}(\zeta)$ [1].
\[ P_L(\zeta) = \sum_{\ell=1}^{2L} c_{\ell} \zeta_{\ell} \]  

where the \( L \) roots \( \zeta_{\ell} \), corresponding to the incidence angles are all within (without) the unit circle in the complex \( \zeta \) plane and

\[ \cos \psi_{\ell} = \frac{\lambda}{2\pi d} \arg(\zeta_{\ell}) \quad \ell = 1, \ldots, L. \]  

**EXPERIMENTAL RESULTS**

The performance of the MRP algorithm was evaluated experimentally using the hardware setup shown on Figure 1. Single channel downconversion with four Mini Circuits ZEM-4300ZH mixers was implemented (thus saving one fast A/D converter per array element) and the complex baseband signal format required by the DF algorithm was synthesized in software with the aid of the Hilbert transform. The image contribution from the adjacent band was cancelled employing a novel recursive sample data covariance matrix subtraction scheme [5] [6]. The LO was generated by the NOVA NS3-17001002 RF signal source and amplified by the Mini Circuits RL-2300 RF amplifier and fed through the 4-way MECA 804-2-1.500V power splitter delivering +10.3dBm to each of the four mixers. Measurements were carried out in the 1910-1915 MHz band using a –15dBm CW signal generated by another NOVA NS3-17001002 RF signal source. The signal was fed to the second 4-way MECA 804-2-1.500V power splitter followed by four Spectrum LS-002-2121 continuously adjustable RF phase shifters emulating the outputs of a linear array of decoupled elements. Power delivered to the mixer RF port was –21dBm. Mixer insertion losses were approximately 6.6dB. IF low pass filters (Mini Circuits SLP-5) provided 40dB attenuation for frequencies higher than 6MHz acting effectively as anti-aliasing filters. Voltage amplifiers (Advanced Receiver Research model P.O.1-30/20VD) that provided 20dB amplification were used to amplify the signals fed to the A/D converters resulting in approximately –8 dBm signal power. The signals were sampled at 25MHz using two Compuscope 1250-1M two-channel 12-bit A/D cards with 10 effective bits thus setting the upper limit on the SNR at almost 60dB.

The phase difference between the channels was set to \( 25^0 \). Assuming that this corresponds to an interelement spacing of \( \lambda/2 \) the equivalent DOA computed from (3) is \( 82.0164^0 \). The DOA estimated by the MRP algorithm was \( 82.0083^0 \) giving an error of \( 0.008^0 \). The record length was 2.6 ms. Figure 2 shows the polar diagram of the two reciprocal roots provided by the MRP algorithm.

![Figure 1. Hardware setup for performance evaluation of the root based DF algorithms.](image-url)
Figure 2. DOA estimation using MRP algorithm. True DOA was $82.0164^0$ while estimated DOA was $82.0083^0$ giving estimation error $0.008^0$.

REFERENCES


