Mathematical model of stochastic algorithm in digital printing
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Keywords: stochastic curves, digital printing, PostScript

Abstract
Generating models of the original stochastic curves are being explored, and their impact on forming the security papers. Differences between PostScript routines are compared regarding limitations of today digital techniques. Discussion is spread on the programming and control of the stochastic vibrations in digital printing. This investigation brings new PostScript routines for versatile halftone screen patterns, as well as suggestions to the development of individual halftone screen shapes.

1. Introduction
Stochastic solution to the individualisation of a print is a marginal part of the digital printing. Computer graphics enable program intervention during printing. First of all, on each printed sheet a new number is feasible, another image or text prepared in separate files. Secondly, individualisation can be generated by an algorithm (usually used in numbering), meaning that data, which does not exist in a computer memory but is created logically according to a program written in C, Pascal or PostScript, is being printed. The highest individualisation levels are random picked line parameters and curves in enclosed group of possible movements or random picked halftone screen elements independently for each pixel.

2. Methods of work
Experiments are based on program solution of assigning random halftone screen shapes to each pixel separately. In order to achieve this, this is created:

1. mathematical models that produce exact shapes of halftone screen elements (Figure 1 - a, b, c, d),
2. procedure which will assign different screen shape and angle to each pixel and
3. PostScript programs which totally realise new solutions to a print individualisation.
This is about pseudorandom numbers with known seed and procedure of generating. That is why it is possible to repeat a series of random numbers, and realisation of a stochastic screened picture. Defining a seed is advised to be putted in function of firm data in the file. For example, if we are dealing with personal document with a photo, a seed can be generated from following data: date of birth and address. Repeating and authenticating a photo could be done only by a person who has full algorithm for a random variable generator. Original PostScript programs and procedures are given as a base to possible further research on stochastic selection of a screen type, frequency and angle on the unique picture.

3. Mathematical models of halftone screen elements

Our basic group contains fifty original shape models of halftone screen elements. Single colour portrait photos on documents usually have 150 kB of memory, so the same screen shape is repeated on an average 3000 times with different level of gray, meaning it appears with a different shape when printing. Figures 1a, 1b, 1c and 1d show four different models, each shown in 12 different levels of gray. Associated mathematical relations and PostScript commands are:

Brighter and darker pixels of the same model are different in a great deal, so one could get the impression of halftone screen elements to be originate from different mathematical relations (exclusively present on a model 1b and 1c).

Model 1 (1) \[ f(x,y) = 1 - \left|\left(\sqrt{|x|} - y\right) / 2\right| \]
\{neg exch abs sqrt add 2 div abs 1 exch sub\} setscreen

Model 2 (2) \[ f(x,y) = 1 - \left|\left(\ln|x + 1.1| - y\right) / 3\right| \]
\{neg exch 1.1 add abs ln add 3 div abs 1 exch sub\} setscreen

Model 3 (3) \[ f(x,y) = 1 - \left|\left|x \cdot 0.5 - |y|\right| / 2\right| \]
\{abs neg exch abs 0.5 mul add 2 div abs 1 exch sub\} setscreen

Model 4 (4) \[ f(x,y) = 1 - \left|\left|x - |y|\right| / 2\right| \]
\{abs neg exch abs add 2 div abs 1 exch sub\} setscreen
Figure 1 - New shapes of halftone screen elements

gsave 10 10 translate
/font1{/FSHelvetica findfont 10 scalefont setfont} def
/squares <2E2952501EA07099506680B8> def
/r1 {neg exch abs sqrt add 2 div abs 1 exch sub} bind def
/r2 {neg exch 1.1 add abs ln add 3 div abs 1 exch sub} bind def
/r3 {abs neg exch abs 0.5 mul add 2 div abs 1 exch sub} bind def
/r4 {abs neg exch abs add 2 div abs 1 exch sub} bind def
/W 120 def /H 90 def /dx 20 def /dy 10 def

%Figure 1a
0 H dy add translate
/font1{-8 H 2 div moveto (a) show
gsave 15 45 {r1} setscreen
W H scale
4 3 8 [4 0 0 3 neg 0 3] {squares} image
restore
%Figure 1c
W neg dx neg add
/font1{-8 H 2 div moveto (c) show
gsave 15 45 {r3} setscreen
W H scale
4 3 8 [4 0 0 3 neg 0 3] {squares} image
restore
%Figure 1b
W dx add 0 translate
/font1{-8 H 2 div moveto (b) show
gsave 15 90 {r2} setscreen
W H scale
4 3 8 [4 0 0 3 neg 0 3] {squares} image
restore
%Figure 1d
W dx add 0 translate
/font1{-8 H 2 div moveto (d) show
gsave 15 90 {r4} setscreen
W H scale
4 3 8 [4 0 0 3 neg 0 3] {squares} image
restore restore showpage

Figure 2 - Source PostScript program for Figure 1
4. Experiments of stochastic solution to halftone screen, ruling and angle

A portrait on figure 3 is solved with 4 halftone screen models (Figure 1). Pixel is enlarged, the number of pixels reduced, and screen ruling also reduced because of better presentation. Four solutions are compared. Standard halftone screen with circle dots and bigger screen ruling (150 dpi) is used as a control pattern (Figure 3a). Total control of the right way or working different screens, that are repeated one after another gives "shaped moiré" with a period of multiplication product in the size of a pixel in the number of different screen models in program (Figure 3b). Moiré disappears if a selection of screen model is done randomly. Except this, screen ruling can also be randomly picked (Figure 3c), as well as the screen ruling together with screen rotation angle (Figure 3d).

This article encounters the original source of routine and procedure continuity in PostScript. Figures 4a, 4b, 4c, and 4d correspond to figures 3a, 3b, 3c and 3d. Change of a seed value (Figure 4c and 4d) will give totally new order of halftone screen solutions. PostScript solution also includes data of a picture, which otherwise in an operating program of the mass usage have to be taken from an external file (series "Grga" from a program on Figure 4a).

Figure 3b uses the same pixel continuity, but in the way of associating each pixel with a different screen algorithm picked out of the algorithm continuity given on figure 1. Program solution becomes denuded, but it is created in a way which can make each researcher interested in it, continue the development of his own algorithms, procedures and finally apply it in specialised usages.

The third solution includes a random variable generator to choose algorithm of a shape of halftone screen element, with screening angle \( \alpha \) equals 0° (Figure 3c and 4c). The seed is set to be fixed (SEED = 12345678). Program shown on Figure 4d generates a contents of Figure 3d, but with a different seed (SEED = 246135). It is also important to say that in this example a random selection of screen \( \alpha \) angle is embedded. A new plan of associating the shape of halftone screen element and screen angle of the same pixel continuity, (that is the same picture) has resulted.
Figure 3. a) standard dot halftone screen
b) screening with cyclic continuity of screening algorithm
c) screening with stochastic selection of halftone screen element shape algorithm (SEED = 12345678) \( \alpha = 0^\circ \)
d) screening with random selection of halftone screen element algorithm and random selection of screening angle \( \alpha \) (SEED = 246135)
Fig. 4. a) program for standard dot halftone screen
b) program with cyclic continuity of screening algorithm
Fig. 4. c) program with stochastic selection of shape of halftone screen element algorithm (SEED = 12345678) $\alpha = 0^\circ$

c) program with random selection of screening element algorithm and random selection of screening angle $\alpha$. (SEED= 246135)
5. Conclusion

In documents and post stamps production with high degree of protection and uniformity, introduction of stochastic individualisation on a pixel screening level is recommended. This solution is possible only on digital graphic prepresses and digital printing directed by PostScript. Recommended solution point out the important of studying the computer graphics and programming.

Stochastic screening models successfully solve the protection of print authenticity, therefore it shall be found everywhere, where it is a subject of high importance. In security program we suggest the shapes of halftone screen element which contain lines and curves (shapes on Figure 1c and 1d). First experiences were on simple trigonometric functions (sin, cos). More sophisticated shapes (Figure 2) provide authenticity and characteristics of the printing house that produces this kind of securities.

Researches with multihalftonecreen shapes and stochastic selection of screen ruling and angle are extended to fullcolour originals. This area has specific usage when solving portraits and stamps. Vibrations, caused by different screening of the same pixel provide unrepeatabillity and strong protection against all scanning techniques.

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