COMPARISON OF THE ROTATING PRANDTL MODEL WITH K (z) AND A MESOSCALE NUMERICAL MODEL

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Abstract: Deployed analytical approach extends Prandtl model for simple katabatic flow by using vertically varying eddy diffusivity and allowing for the Earth rotation. The related flow pertains to long glaciers on Greenland and Antarctic, which are poorly resolved in climate and some numerical weather prediction (NWP) models. We compare the extended 1D analytical Prandtl model for (U, V, θ) with a mesoscale numerical model. The MIUU mesoscale model employs a fine vertical resolution and a higher-order turbulence parameterization scheme. The analytical model slightly underestimates both the height and the speed of U maximum, and the variations in θ. However, it successfully describes the time evolution of the V component and its maximum, although somewhat overestimating the magnitude of V. Nevertheless, the overall tentative agreement of the analytical model with the MIUU model profiles is good. The differences arise from the hierarchy of the effects of the various assumptions made in the extended analytical Prandtl model. Hence, further improvements of the Prandtl model will be amendable as well as recommendations for a better parameterization of katabatic flows in large-scale numerical models.

Keywords: *Stable boundary layer, Prandtl model, MIUU model*

1. INTRODUCTION

Katabatic flows are downslope, buoyancy driven, flows that develop over inclined radiatively cooled surfaces regular in the stable atmospheric boundary layer (ABL). The ubiquitous nature of katabatic flows over e.g. Antarctica and Greenland, not to mention smaller areas such as Iceland, and their cumulative effects, implies that the katabatic wind contributes to the general circulation (Parish and Bromwich, 1991).

The detailed structure of katabatic flow still remains an important modelling issue (e.g. Weng and Taylor, 2003). The stably stratified boundary layer is usually poorly resolved in many numerical models (e.g. Zilitinkevich et al., 2006). In order to capture the pronounced shallow low-level jet (LLJ) and sharp near-surface vertical temperature gradient that characterize the pure katabatic flows (e.g. Grisogono and Oerlemans, 2001; Van den Broeke et al., 2002), a sufficient vertical resolution has to be used (e.g. Renfrew, 2004). Ever increasing resolution of the NWP and various regional models calls for continuous and necessary improvements of current parameterizations (e.g. various corrections to the Monin-Obukhov length, etc.).

A simple Prandtl model of katabatic flows represents a balance between the negative buoyancy production due to the surface potential temperature deficit and dissipation by turbulent fluxes (e.g. Mahrt, 1982; Egger, 1990). Stiperski et al. (2007) extended the Prandtl model by including the Coriolis force in order to be able to cover long polar slopes and the corresponding long-lived strongly stable ABL. Kavčič and Grisogono (2007) introduced gradually varying eddy diffusivity in the analytical model of Stiperski et al. (2007).

Here we compare the analytical solutions for katabatic flows from Kavčič and Grisogono (2007) and the corresponding simple 1D numerical solutions with MIUU numerical mesoscale model (e.g. Andrén, 1990; Grisogono and Enger, 2004), revoking a known suggestion that an additional alternative for surface-layer scaling may be invoked – that from the Prandtl model relating to the LLJ height (Munro, 2004; Grisogono and Oerlemans, 2001). The emphasis is put on the time development of the main components of the improved Prandtl model: the down- and the cross-slope flow component, U and V, and the potential temperature perturbation θ.

2. ANALYTICAL SOLUTIONS AND MIUU MESOSCALE MODEL

2.1 The extended Prandtl model

The rotating Prandtl model describes a hydrostatic, linear, one-dimensional Boussinesq flow with the effects of the Coriolis force included (Stiperski et al., 2007). As in the classical Prandtl model (e.g. Mahrt, 1982; Egger, 1990; Parmhed et al., 2004), the *K*-theory is invoked to model the turbulent fluxes. Kavčič and Grisogono (2007) obtained solutions of the rotating model for the case of non-constant *K* by using the WKB method (after Wentzel, Kramers and Brillouin, who popularized the method in theoretical physics) of zero-order (e.g. Bender and Orszag, 1978). We briefly list the WKB solutions for the the potential temperature perturbation θ and the downslope and cross-slope components of the wind vector (U, V):

$$\theta_{WKB}(z) = C\exp(-s(z))\cos(s(z)), \quad s(z) = \frac{\sigma_{WKB}(z)}{\sqrt{2}} \tag{1}$$

$$U_{WKB}(z) = \frac{C\sigma_0^{\;2}}{\gamma\sin(\alpha)}\exp(-s(z))\sin(s(z)), \tag{2}$$

$$V_{WKB}(z,t) = \frac{Cf\cot(\alpha)}{\Pr\gamma}\left[1 - erf\left(\frac{I(z)}{2\sqrt{t\Pr}}\right) - \exp(-s(z))\cos(s(z))\right], \tag{3}$$

$$\sigma_{WKB}(z) = \sigma_0 I(z),\ \sigma_0^{\;4} = \frac{N^2\Pr\sin^2(\alpha) + f^2\cos^2(\alpha)}{\Pr^2},\ \ I(z) = \int_0^z K(z)^{-1/2}dz. \tag{4}$$

Here the z axis is perpendicular to the surface (x axis) sloped with the negative (clockwise) angle α from the horizontal; t is the time. The symbols have their usual meaning: K is eddy diffusivity, θ_0 is a reference potential temperature, f is the Coriolis parameter, g is acceleration due to gravity, Pr is the turbulent Prandtl number, $N^2 = g\gamma/\theta_0$ is the buoyancy (Brunt-Vaisala) frequency and $C < 0$ is the constant surface-potential-temperature deficit (total minus the background potential temperature), applied to an undisturbed atmosphere-surface interface. Slope angle α, for which the katabatic wind is successfully treated by the model, typically does not exceed ~ 10°, therefore giving a reasonable assumption of using the constant gradient of the background potential temperature γ in the true vertical. The WKB solutions (1), (2) and (3) hold after a typical period $T=2\pi/(N\sin(\alpha))$ (Mahrt, 1982; Grisogono, 2003). For further details see Kavčič and Grisogono (2007) or Stiperski et al. (2007).

There are various choices of the *K*-profile possible (see Grisogono and Oerlemans, 2001). Here we apply the modelled *K*-profile, calculated with MIUU model, with the $K_{max} \approx 4.3$ m^2 s^{-1} at height $h_{max} \approx 20$ m. The $K(z)$ decreases almost linearly with height after its maximum, reaching zero at $z \sim 1000$ m.

2.2 Mesoscale model MIUU

The MIUU model is a nonlinear, 3D, hydrostatic, constant *f*-plane model with a higher-order closure turbulence parameterization scheme (Andrén, 1990; Tjernström and Grisogono, 2000; Grisogono and Enger, 2004). It solves five prognostic equations (and many diagnostic equations each time step): for the two horizontal wind components, for the potential temperature (Θ), specific humidity and turbulent kinetic energy (TKE).

The model setup is: 0.9 m < Δz < 30 m, $\Delta x \approx 3$ km gradually stretched toward lateral boundaries, $\Delta x \leq 28$ km, with total 89 × 7 × 250 grid points (no y-variations). Time step is $\Delta t = 12$ s and the model top is $z_{TOP} = 7.8$ km with a sponge starting at $z = 4.5$ km. The five first and last lateral grid points use a first-order forward-upwind advection scheme on which a ”constant inflow–gradient outflow” condition is applied (e.g. Grisogono and Enger, 2004). Furthermore, a weak additional “lateral diffusion” is used. For the upper part of the sponge layer the same dissipative first-order advection scheme is employed instead of O((Δx)^3, (Δt)^3) as default (Enger and Grisogono, 1998). The model levels follow the terrain, gradually leveling off horizontally with height. An abrupt initialization starts with the given terrain and the idealized atmospheric profiles.

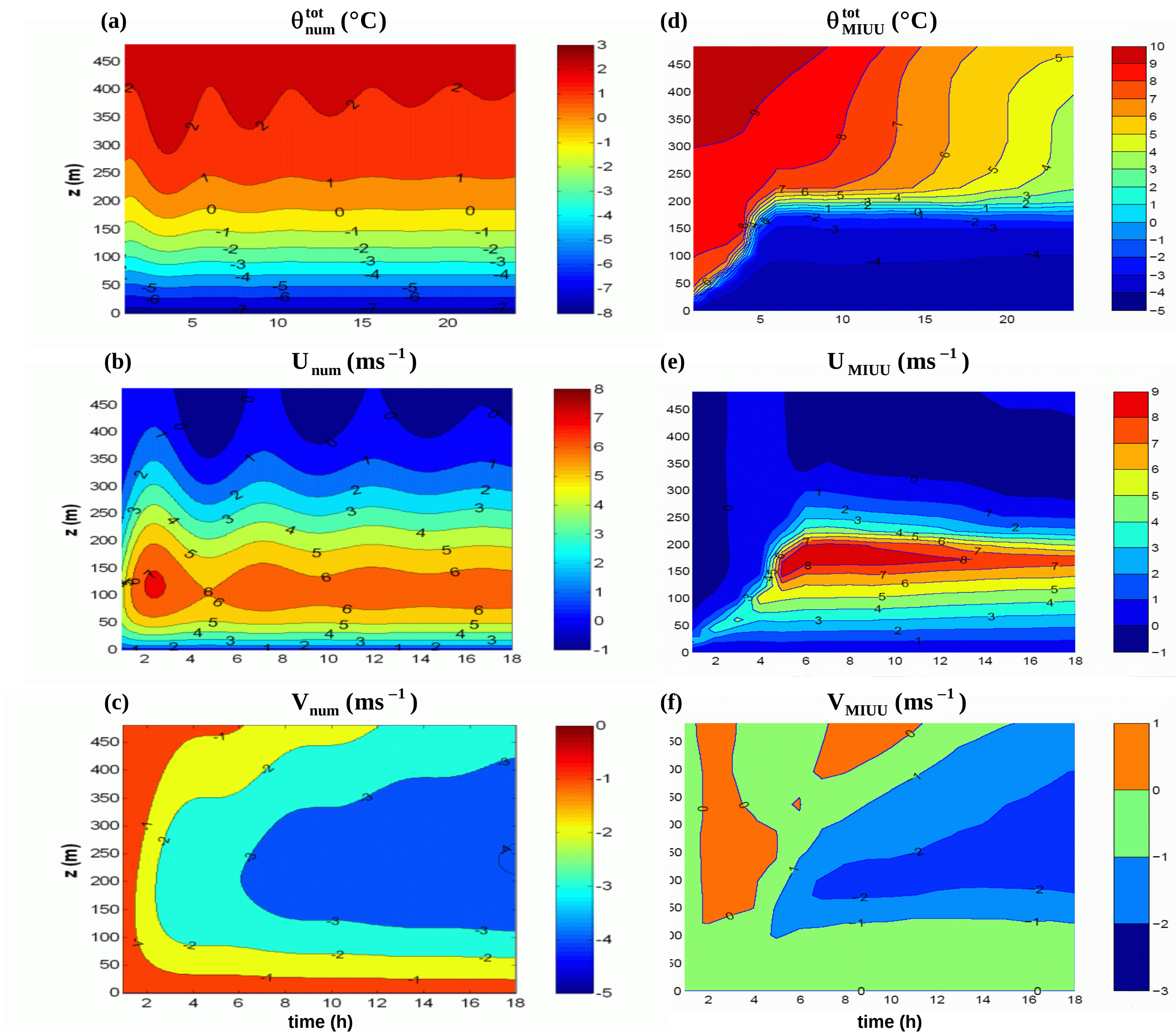

Figure 1: Comparison of solutions of the simple time-dependent numerical model solutions, left, for (a) $\theta^{tot}{}_{num}$, (b)U_{num} and (c) V_{num} (similar to the WKB solutions (1), (2) and (3)) with the corresponding MIUU profiles, right, for (d) $\theta^{tot}{}_{MIUU}$ (e) U_{MIUU} and (f) V_{MIUU}. Here *K(z)* is from MIUU model with $K_{max} \approx 4.3$ m^2 s^{-1} at $h_{max} \approx 20$ m. Other parameters are (f, α, γ, Pr, C) = (1.03×10^{-4} s^{-1}, $-1.5°$, 5×10^{-3} K m^{-1}, 1.1, -8°C). $\theta^{tot}{}_{num} = \theta_{num} + \gamma z$, where θ_{num} is the potential temperature perturbation in the simple numerical model (the same for the WKB solution, Eq. 1). The simple numerical model top is at 2000 m.

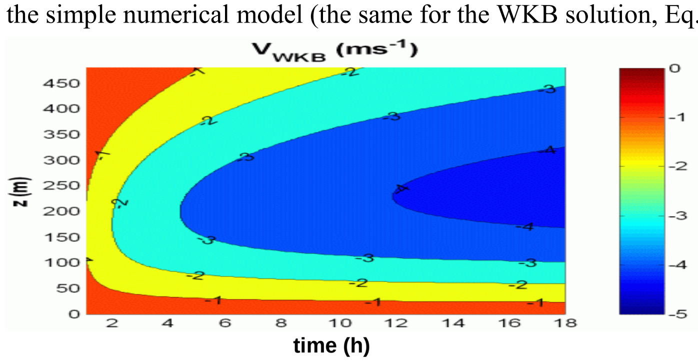

Figure 2: The WKB solution for V_{WKB} (Eq. 3*)*. U_{WKB} (Eq. 2) and $\theta^{tot}{}_{WKB} = \theta_{WKB} + \gamma z$, with θ_{WKB} given in (1), are steady and thus not shown here. The rest as in Figure 1, thus compare it with Fig. 1c and 1f. The WKB solutions hold after t ≈ T = 4.98 h.

3. DISCUSSION AND CONCLUSIONS

Analytical WKB solutions (1), (2) and (3) correspond to the time-dependent solutions obtained by the simple 1D numerical model (θ^{tot}_{num}, U_{num}, V_{num}) after $t \approx T$ = 4.98 h, e.g. compare V_{num}, Fig. 1c, and V_{WKB}, Fig. 2 (e.g. Stiperski et al., 2007; Kavčič and Grisogono, 2007). Thus, it is reasonable to use the simple numerical solutions (θ^{tot}_{num}, U_{num}), Fig. 1a and 1b, respectively, for the comparison with MIUU model profiles (θ^{tot}_{MIUU}, U_{MIUU}), Fig. 1d and 1e, respectively. Here $\theta^{tot}_{num} = \theta_{num} + \gamma z$. The setup of the simple numerical model is adjusted to the parameters used in MIUU model, that is: (f, α, γ, Pr, C) = (1.1×10^{-4} s^{-1}, −1.5°, 5×10^{-3} K m^{-1}, 1.1, −8°C).

When comparing the U profiles in Fig. 1b and 1e, one sees that U_{num}, corresponding to U_{WKB} after $t \approx T$, slightly underestimates both the height and the speed of the U maximum in MIUU model, but describes well the time evolution of U_{MIUU}. This is probably because of employment of the coarser vertical resolution and a higher-order turbulence parameterization scheme in MIUU model, resulting in greater values of TKE that result in more mixing and lifting of the U maximum. The variations of θ^{tot}_{MIUU} (Fig. 1d) are also underestimated (Fig. 1a), by θ^{tot}_{num} corresponding to θ^{tot}_{WKB} after $t \approx T$. Nevertheless, the WKB solution for V, Fig. 2, successfully describes the overall time evolution of the V component in MIUU model (V_{MIUU}, Fig. 1f) and its maximum, although somewhat overestimating the magnitude of V_{MIUU}. The differences between the WKB solutions (1), (2), (3) and the simple numerical model on one side, and the MIUU model solutions stem from the effects of the various assumptions made in the extended, but still analytical model. With the further development of the analytical Prandtl model and modifications of the varying eddy diffusivity profile, the WKB solutions may be useful for various model comparisons and for improving current surface–flux parameterizations in climate models.

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