HYDROSTATICS OF A DEFLECTED SHIP HULL

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Approximate terms based on waterplane area characteristics are provided for easy and fast assessment of the effects of deflections of an elastic ship hull on draughts, displacement and still water hull girder loads. A numerically efficient iterative method based on repeated standard rigid girder calculations is proposed for the determination of draughts, displacement and hull girder loads of an elastic ship hull on an optional floating line.

The relevance of both of the methods is empirically justified.

1. Introduction

The paper presents first a comprehensive method for assessment of the deflection effects of an elastic ship hull, on draughts and displacement, as well as on the still water hull girder loads. The method is based on the data observed on board, and uses a parabolic approximation of the deflection line as most of the methods do, e.g. Ref. [1], but taking into account the ship hull form, what the existing methods do not.

Next, an iterative method for hydrostatics and hull girder loads determination of the deflected ship hull on an optional floating line, based on a repeated application of standard rigid body calculation of shear forces and bending moments, e.g. Ref. [2], is presented. Finally, the methods are tested on two ships: "A" - a large OBO-carrier and "B" - a river barge.

2. Approximate assessment of the hull deflection effects

The following considerations are based on the data available on board. The draughts
observed on ship side(s) aft, amidships and forward, see Figure 1, are denoted as follows:

\[ d_{m}^{a}, d_{m}^{f}, d_{m}^{o} \]

The observed amidships draught \( d_{m}^{a} \) corresponds to the "observed" displacement \( \Delta_{a} \), as determined from standard hydrostatic data, see Figure 2.

The standard amidships draught \( d_{m}^{o} \) used for conventional hydrostatic calculations is obtained from the observed draughts as follows:

\[ d_{m}^{o} = \frac{d_{m}^{a} + d_{m}^{f}}{2} \]  

(1)

The "standard" displacement \( \Delta_{a} \) for a ship hull considered as a rigid girder can be obtained from standard hydrostatic data using the standard amidships draught \( d_{m}^{o} \) from eqn. (1).

2.1. Assessment of the hull deflection

The ship hull deflection \( w_{m} \) is usually considered only amidships as the deviation of the observed amidships draught of the standard draught amidships in eqn. (1), as shown:

\[ w_{m} = d_{m}^{a} - d_{m}^{o} = d_{m}^{f} - \frac{d_{m}^{o} + d_{m}^{f}}{2} \]  

(2)

It follows from eqn. (2), that the deflection amidships is positive for a sagging condition.

The second order symmetric parabola, can be used as an approximation for the deflection line of the hull, given in the following form:

\[ w(x) = \frac{w_{m}}{L_{ew}^2} x^2 \]  

(3)

Table I presents a comparison between the parabolic approximation and the theoretically calculated deflection line for the sagging condition of test ship "A", normalized to unity.
Figure 1. Notations for description of a deflected ship hull.
Figure 2: Relations of draughts and corresponding displacements of a deflected ship hull.
Table 1. Calculated and parabolic approximation of a deflection line normalized to unity, for a sagging condition of the test ship "A".  

<table>
<thead>
<tr>
<th>x</th>
<th>w(calc)</th>
<th>w(parab)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>0.4</td>
<td>0.65</td>
<td>0.65</td>
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<tr>
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<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>0.8</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>1.0</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>1.2</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>1.4</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>1.6</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>1.8</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>2.0</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

It is obvious from Table 1 that the calculated deflection line is neither symmetrical nor parabolic, but the deviations of the parabolic form are of limited order. On the other hand, it is not practical to determine the deflection shape on board more precisely.

2.2. Assessment of the draughts and displacement of a deflected ship hull

The hydrostatic characteristics of the ship hull for small deflections, depend mostly on waterplane properties, as can be seen from Figure 1. The longitudinal change in buoyancy due to hull deflection \( w(x) \) for hypothetically parallel ship sides, can be defined at any abscissa \( x \), using the waterplane breadth \( b(x) \), as the change of the sectional area:

\[
a_s(x) = w(x)b(x)
\]

(4)

The decrease and increase of the displacements relative to the "observed" displacement \( \Delta \), of the deflected hull in sagging and hogging conditions respectively, in water of specific gravity \( \gamma \) supposing a symmetric parabolic deflection shape as in eqn. (3), is obtained by lengthwise integration of eqn. (4) as follows:

\[
\Delta_s^a = \gamma \int a_s(x)dx = \frac{w_n\gamma}{(L_m/2)^2} \int b(x)^2dx = \frac{w_n\gamma A_d}{(L_m/2)^2} = w_n\gamma A_d C_d
\]

(5a)

The change of the displacement relative to the "standard" displacement can be expressed as

\[
\Delta_s^a = w_n\gamma A_d - \Delta_s^a = w_n\gamma A_d (1 - C_d)
\]

(5b)

The following symbols are used:
Hydrostatics of a deflected ship hull

\( I_n \) longitudinal moment of inertia of the waterline plane about a transverse axis through the mid of its length; it can be obtained from standard hydrostatic data as follows:

\[
I_n = A_{w1} L_{w1}/2 \cdot \frac{C_L}{C} = I_s + x_{sl}^2 A_{w1} = M_1 L_{w1} \quad \text{(6)}
\]

\( I_s \) longitudinal moment of inertia of the waterline plane about a transverse axis through the center of flotation;

\( A_{w1} \) area of the waterplane of length \( L_{w1} \) and breadth \( B_{w1} \); \( A_{w1} = L_{w1} B_{w1} \)

\( x_{sl} \) distance from the waterline center of gravity to the mid of length \( L_{w1} \);

\( M_1 \) the moment to change trim 1 meter;

\( C_{w1} \) waterplane area coefficient;

\( C_d \) the draught and displacement correction factor in eqn. (5) due to deflection is defined as shown:

\[
C_d = \frac{I_n}{I_s} = \frac{M_1}{A_{w1} x_{sl}^2} \quad \text{(7)}
\]

The correction of the observed amidships draught due to deflection can be determined as a parallel emersion or immersion for sagging or hogging conditions respectively, as follows:

\[
d_{sl}^{\sigma} = \frac{\Delta_d}{A_{w1}} = \frac{w_{sl} C_d}{A_{w1}} \quad \text{(8a)}
\]

The correction of the standard amidships draught can be easily obtained as shown:

\[
d_{sl}^{\sigma} = w_{sl} - d_{sl}^{\sigma} = w_{sl}(1 - C_d) \quad \text{(8b)}
\]

To obtain the actual displacement \( \Delta \) of the ship from the standard hydrostatic data, the equivalent amidships draught can be used, as clarified on Figure 2. Either the observed amidships draught \( d_{sl} \) or the standard amidships draught \( d_{sl}^{\sigma} \) can be corrected to the equivalent amidships draught \( d_{sl}' \) using deflections amidships \( w_{sl} \) from eqn. (2) and the draught and displacement correction factor \( C_d \) from eqn. (7) as shown:

\[
d_{sl}' = d_{sl}^{\sigma} - d_{sl}^{\sigma} = d_{sl}^{\sigma} - w_{sl} C_d = d_{sl}^{\sigma} + d_{sl}^{\sigma} = d_{sl}' - w_{sl}(1 - C_d) \quad \text{(9)}
\]

The actual displacement can be obtained directly from the standard hydrostatic data (displacement curve), using the equivalent amidships draught \( d_{sl}' \) (see Figure 2).
Hence, the actual displacement can be determined relative either to the "observed" displacement $\Delta_o$ or to the "standard" displacement $\Delta_s$ according to eqn. (5) (see Figure 2) as shown:

$$\Delta = \Delta_o - \Delta_s = \Delta_o - w_o A_s C_d = \Delta_o + \Delta_s = \Delta_o + w_o A_s (1 - C_d)$$  \hspace{1cm} (10)

The expressions $A_s C_d$ and $A_s (1 - C_d)$ in eqn. (10) represents the volume of displacement correction per unit of deflection of the ship hull, relatively to the "observed" displacement and "standard" displacement, respectively.

For the test ship "A", hydrostatic data, correction factors $C_d$ determined according to eqns. (7), as well as the values for $A_s C_d$ and $A_s (1 - C_d)$ are given in Table 2.

2.3. Approximation of the waterplane shape

For simplicity, the true waterplane shape will be approximated by a symmetric general parabola of order $k$:

$$b(x) = B_a \left[ 1 - \frac{x^2}{(L_a/2)^2} \right]$$  \hspace{1cm} (11)

In that case, the following relations based on eqn. (11) can easily be proved:

$$k = \frac{C_{my}^a}{1 - C_{my}^a}$$  \hspace{1cm} (12)

$$A_{sl} = \int_{-L_a/2}^{L_a/2} b(x) dx = B_a L_a \frac{x}{k} \frac{1}{1 - \frac{x^2}{(L_a/2)^2}} = B_a L_a C_{my}$$  \hspace{1cm} (13)

$$I_{my} = \int_{-L_a/2}^{L_a/2} b(x) x^2 dx = A_{sl} (L_a/2)^2 C_d$$  \hspace{1cm} (14)

$$C_d = \frac{1}{6C_{my}^a}$$  \hspace{1cm} (15)

For some other types of vessels, e.g. barges or pontoons, a more appropriate approximation of the waterplane shape can be of the linear form given as follows:
Hydrostatics of a deflected ship hull

\[ h(x) = R_0 \left[ 1 - \frac{1 - \frac{C_{WP}}{L_{ref}/2}}{C_{WP}} \right] \]  

Equation (16)

The draught and displacement correction factor for these linear waterplane forms according to eqn. (16), can be obtained as:

\[ C_d = \frac{C_{WP} - \frac{1}{2}}{2C_{WP}} \]  

Equation (17)

2.4. **Assessment of the bending moments and the shear forces**

The standard still water calculations provide draughts, displacement, shear forces and bending moments for the ship hull considered as a rigid girder. The deflection line calculation can be performed using the ship hull girder vertical sectional moments of inertia. The deflection of the ship hull changes the hydrostatic loads and in reverse it affects the draughts, the shear forces and the bending moments. A second order symmetric parabola in eqn. (3), shifted vertically (in order to assure hydrostatic balance) over the parallel immersion \( w_m C_d \), can be used as an approximation for lengthwise distribution of the changes of hydrostatic loads relatively to the ship hull considered as rigid girder:

\[ a(x) = w(x) - w_m C_d = w_m \left[ \frac{-x^2}{L_{ref}^2} - C_d \right] \]  

Equation (18)

The longitudinal change in buoyancy (positive direction upward \( \uparrow \)) for hypothetically parallel ship sides due to hull deflection \( a(x) \), relatively to the floating line of a hypothetically rigid ship hull, can be obtained on the basis of eqn. (18), as follows:

\[ q_B = -a(x) h(x) \]  

Equation (19)

The maximum changes of the shear forces due to the deflection relative to the rigid body calculation occur at the positions \( \pm x_q \) where \( q_B(x_q) = 0 \) in eqn. (19), as follows:

\[ x_q = \pm \frac{L_{ref}}{2} \pm \frac{x}{L_{ref}} C_m \]  

Equation (20)

and the amount of this maximum in shear force change is as shown:
\[
Q_d = \gamma \int_{-l_c}^{l_c} q_d(x) \, dx = 2w_cR_d \int_{-l_c}^{l_c} \psi \, dx
\]

(21)

The change of the amidships bending moment due to deflection relative to the rigid body calculation is obtained by lengthwise integration applied on eqn. (19), as shown:

\[
M_d = \gamma \int_{-l_c}^{l_c} q_d(x) \, dx = w_cR_d \int_{-l_c}^{l_c} \psi \, dx
\]

(22)

The shear force correction factor \(C_M\) in eqn. (21) based on eqn. (11) is expressed as:

\[
C_M = C_d \left[ \frac{1}{3} - 3(1 - C_M^2)^2 \right] - \frac{C_M}{1 + C_M^2}
\]

(23)

It is easily recognizable that the bending moments calculated for the conventional ships considered as rigid girders, are generally reduced when the resulting deflection of the hull is taken into account. The amidships bending moment correction factor \(C_M\) in eqn. (22) based on general parabola assumption in eqn. (11), is expressed as:

\[
C_M = C_d \frac{6C_M^2 - 15C_M + 10}{6C_M^2 + 29C_M^2 - 46C_M + 24}
\]

(24)

The shear force and amidships bending moment correction factors based on linear approximation of the waterplane shape in eqn (16), can be expressed as:

\[
C_d = C_d^0 \left[ \frac{1}{3} - 3(1 - C_M^2)C_d^{02} \right]
\]

(25)

\[
C_M = \frac{12C_M^2 + 8C_M - 5}{-72C_M^2}
\]

(26)

For faster assessments of deflection effects, the linearized draught, amidships bending moment and shear force correction factors are given as follows (see also Figure 3):
Figure 3. Correction factors based on $C_{WP}$ for draughts $C_d$ for shear forces $C_Q$ and for bending moments $C_M$
- the lower curves correspond to general parabolic approximation of the waterline (hairline);
- the upper curves correspond to linear approximation of the waterline (dash line);
- the lines between correspond to linearized correction factors (dash and dot line).
\[ C'_d = \frac{1}{2} C_{WP} \]  
(27)

\[ C'_d = 0.08988 C_{WP} - 0.02573 \]  
(28)

\[ C'_{CM} = \frac{1}{22} (22 C_{WP} - 7) \]  
(29)

The correction factors \( C_d \), \( C_D \), \( C_M \) and \( C_{WP} \) for the test ship "A" based on eqns. (15), (23), (24) and (20) respectively, are given in last four columns in Table 2.

**Table 2: Hydrostatic data and correction factors for the test ship "A".**

<table>
<thead>
<tr>
<th>( d [\text{m}] )</th>
<th>( C_{WP} )</th>
<th>( I_{11} [\text{m}^4] )</th>
<th>( A_{p1} [\text{m}^2] )</th>
<th>( I_1 [\text{m}^4] )</th>
<th>( M_c [\text{m}^3] )</th>
<th>( A_{wp} C_d )</th>
<th>( A_{wp} (1 - C_d) )</th>
<th>( C_d )</th>
<th>( C_D )</th>
<th>( C_M )</th>
<th>( C_{WP} )</th>
<th>( C'_d )</th>
<th>( C'_{Dd} )</th>
<th>( C'_{CM} )</th>
<th>( C'_{WP} )</th>
</tr>
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<tbody>
<tr>
<td>Actual ship data, OHO-carryer</td>
<td>Parabolic deflection</td>
<td>Waterline approximation</td>
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<td></td>
<td></td>
<td></td>
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<td>( 9 )</td>
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<td>.053</td>
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<td>137491</td>
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<td>.068</td>
<td>.272</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

The factors \( C_d \) in eqns. (15), (17), (27), \( C_D \) in eqns. (23, 25, 28) and \( C_M \) in eqns. (24, 26, 29) depend only on the waterplane area coefficient \( C_{WP} \), and are represented on Figure 3.

It is obvious from Figure 3, that the usually applied 'quarter mean draught' correction, e.g. Ref. [11], corresponding to \( C_d = 1/4 \), is valid only for a specific waterplane coefficient of about \( C_{WP} \approx 0.834 \) for general parabolic approximation of the waterline. For other values of \( C_{WP} \), the correction differs significantly from one quarter, that confirms the need for more precise approximation of the hull form, i.e. of the waterplane shape.

3. Iterative calculation of the hull deflection effects

The geometrical characteristics of a deflected ship hull on the optional floating line \( d(x) \) at position \( x_n \), (see Figure 4) can be defined by correction of the standard draughts \( d(x_n) \) of the hypothetically undeformed hull, in successive iterations, for an amount of deflection \( w(x_n) \) determined by a standard calculation method in previous iteration, as follows:
\[ d(x) = d'(x) + w(x) \]  
(30)

The sectional area \( A_t \) to draught \( d(x) \) of a deflected hull is then as shown:

\[ A_t(x, d(x)) = A_0(x, d'(x)) + w(x) \]  
(31)

The correction in eqns. (30) and (31) is not applied when the bottom is cut, or when the deck is immersed.

The geometry of a deflected hull can be described by eqns. (30) and (31), using standard data for a non-deflected hull (Bonjean's curves), and the standard calculation methods can be applied by iterations, as presented by a flowchart in Figure 5. Convergence criteria can be based on the relative changes in the bending moments or in the deflection line. Sufficient accuracy will normally be achieved in 3-5 iterations. The first iteration corresponds to the standard rigid body calculation, taking the initial hull deflection into account, if any.

The draughts obtained for the rigid ship hull can be regarded as the equivalent draughts with respect to the draughts of the deflected hull, as it is considered earlier.
4. Examples

Two test vessels: “A” and “B” are investigated, with principal data and loading conditions according to Tables 3, and 4.

Table 3. Principle data of test ships “A” and “B”.

<table>
<thead>
<tr>
<th>Ship</th>
<th>( L_{50} ) [m]</th>
<th>( L_{B} ) [m]</th>
<th>( B ) [m]</th>
<th>( D ) [m]</th>
<th>( d ) [m]</th>
<th>( A ) [t]</th>
<th>( LS ) [t]</th>
<th>( I_{ww} ) [m³]</th>
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</thead>
<tbody>
<tr>
<td>“A”</td>
<td>231.00</td>
<td>243.00</td>
<td>11.25</td>
<td>20.60</td>
<td>15.00</td>
<td>101948</td>
<td>18217</td>
<td>307.28</td>
</tr>
<tr>
<td>“B”</td>
<td>27.00</td>
<td>27.00</td>
<td>11.00</td>
<td>4.50</td>
<td>3.30</td>
<td>2650</td>
<td>650</td>
<td>1.64</td>
</tr>
</tbody>
</table>

Table 4. Load cases for test ships “A” and “B”.

<table>
<thead>
<tr>
<th>Ship</th>
<th>Load case 1</th>
<th>Load case 2</th>
<th>Load case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>“A”</td>
<td>lightship (LS)</td>
<td>under service, fully loaded</td>
<td>light ballast, 100 % stores</td>
</tr>
<tr>
<td>“B”</td>
<td>ballast condition</td>
<td>2000 t bulk cargo</td>
<td>600 t heavy cargo</td>
</tr>
</tbody>
</table>

First, a standard calculation of hydrostatics, shear forces and bending moments for the
Hydrostatics of a deflected ship hull

ship hull as a rigid body is performed. Next, the calculation for the ship hull considered as an elastic body is performed by iterations, taking the deflection line into account until the convergence is achieved, using the procedure described in section two.

Finally, approximate terms as presented in section one, are applied. The results are presented in Table 5.

Table 5. Displacements, deflections, draughts, bending moments and shear forces:
- (1st row) calculated for the ship hull as a rigid body;
- (2nd row) calculated for elastic ship hull using iterative procedure;
- (3rd row) assessed by the approximate term.

<table>
<thead>
<tr>
<th>CASE</th>
<th>Δ</th>
<th>ΔW</th>
<th>ΔW'</th>
<th>ΔM</th>
<th>ΔM'</th>
<th>ΔSP</th>
<th>BM</th>
<th>M'</th>
<th>SF</th>
<th>Qd</th>
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<td>95</td>
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5. Conclusions

Instead of the usually applied "quarter mean draught" correction, the presented terms based on the observed draughts, the parabolic approximation of the deflection line of
the ship hull and the true shape of the waterline, provide a fast practical method for
determination of draughts, displacement, shear forces and bending moments of a
deflected ship hull.

It is demonstrated that the deviations due to the parabolic form of the theoretical
deflection line are in general of limited order for conventional ships. The presented
expressions based on the application of the standard hydrostatic data as the moment to
change trim one meter, the waterplane longitudinal moment of inertia, the waterplane
area, can easily be used on-board.

The iterative application of a standard rigid-body calculation procedure for an
optional floating line, as presented in the paper, allows theoretical determination of
the ship hull deflection effect on draughts, displacement, shear forces and bending
moments, in a limited number of iterations (3 to 5).

The approximate terms based only on the waterplane area coefficients, are provided
for better assessments and are in a good agreement with the results of the iterative
procedure.

The maximum bending moments of a deflected ship are in general slightly less
(approximately 2%) than the bending moments of a ship hull considered as a rigid
body. The shear forces of the deflected ship are also slightly changed compared to the shear
forces determined for the ship as a rigid girder (approximately 1%).

More accurate and practical corrections of draughts and displacement due to
deflection as demonstrated in the paper, can be beneficial in ship service.

References

1957.

Table of notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A_{wl}$</td>
<td>area of the water plane;</td>
</tr>
<tr>
<td>$A_s$</td>
<td>sectional area;</td>
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<tr>
<td>$BM$</td>
<td>bending moments;</td>
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<tr>
<td>$B_{wl}$</td>
<td>waterline beam;</td>
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<td>$C_d$</td>
<td>correction factor for draught and displacement of a deflected hull relative to the observed amidships draught;</td>
</tr>
<tr>
<td>$CM$</td>
<td>correction factor for the bending moment amidships of a deflected hull relative to the value calculated for a hull as a rigid body;</td>
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</table>
Correction factor for the shear force at the position \( x_{m} \) of a deflected hull relative to the value calculated for a hull as a rigid body;

Correction factor for the lengthwise position of the maximum change in shear force;

Waterline coefficient;

Observed draughts aft, amidships and forward;

Change of the observed amidships draught due to the hull deflection;

Change of the standard amidships draught due to the hull deflection;

Equivalent amidships draught corresponding to the actual displacement;

Standard amidships draught;

Moment of inertia of the waterline about a transverse axis through the mid of length;

Sectional moment of inertia;

Moment of inertia of the waterline about a transverse axis through the center of flotation;

Waterline length;

Change of the bending moment amidships for a deflected hull relative to the value calculated for a hull as a rigid body;

The moment to change trim 1 meter;

Change of the shear force at position \( x_{m} \) for a deflected hull relative to the value calculated for a hull as a rigid body;

Shear forces;

Positions of maximal changes of shear forces;

Distance from the waterline center of gravity to the mid of length \( L_{m} \);

Hull deflection amidships (maximal deflection), considered as positive for hogging;

Specific gravity of water;

Actual displacement;

“Observed” displacement corresponding to the observed amidships draught;

“Standard” displacement corresponding to the standard amidships draught;

Change of the displacement for deflected hull relative to the “observed” displacement;

Change of the displacement for deflected hull relative to the “standard” displacement;