MAGNETIC FIELD IN THE VICINITY OF DISTRIBUTION TRANSFORMERS

SUMMARY

A quasi-static analysis of an oil-type distribution transformer was described to determine the ambient magnetic field in the surrounding region. Windings and terminals were assumed to be primary sources of the magnetic field. Core and tank of the transformer were modeled by secondary sources (surface current density, surface charge density, and eddy current density). In order to find these sources, the system of integral equations was transformed into a linear system. Magnetic field was calculated by evaluating volume and surface integrals. The calculated and measured results of the magnetic induction were compared on a 630 kVA transformer. Evaluation of the magnetic induction was made for distribution transformers in the range 50-2500 kVA.

Key words: distribution transformer, magnetic field, magnetic induction, integral equations

1. INTRODUCTION

The demand for a regulation of electromagnetic field limits resulted from the public concern about the electromagnetic emissions. The ICNIRP guidelines [1] for limiting exposure to time-varying, magnetic and electromagnetic fields have been widely accepted and implemented in many countries. According to them, magnetic induction reference level at 50 Hz is 500 μT for the occupational exposure, and 100 μT for the public exposure. All values are unperturbed rms. However, some countries have prescribed lower limit values referring to the precautionary principle. Switzerland has limit of 1 μT [2], Italy of 3 μT, Slovenia of 10 μT, Croatia of 40 μT [3], etc. Oil-type distribution transformers are used in urban areas. As a part of power distribution system, they emit the electromagnetic field. In this paper 1 μT and 40 μT curves are determined in the surrounding region of such transformers at 50 Hz. The only source of the magnetic field is assumed to be transformer itself.

These curves can be determined by measuring or computing [4]. The measurement of the magnetic field is time consuming. Also, special equipment and the space with no other sources are required. Therefore a numerical computation technique was presented to evaluate the magnetic field of distribution transformer.

2. DISTRIBUTION TRANSFORMERS

When talking about oil-type distribution transformers in this paper, we mean on all three-phase oil-immersed transformers from 50 kVA to 2500 kVA with the highest voltage for equipment not exceeding 36 kV and with rated secondary voltage 400-433 V [5]. The analysis of magnetic field was made for the typical 630 kVA oil-type transformer (table I).
Table I. Transformer technical data.

<table>
<thead>
<tr>
<th>Type</th>
<th>Oil-immersed, three-phase distribution transformer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>630 kVA</td>
</tr>
<tr>
<td>Rated voltages</td>
<td>20 kV / 420 V</td>
</tr>
<tr>
<td>Vector group</td>
<td>Dyn5</td>
</tr>
<tr>
<td>Impedance voltage</td>
<td>4 %</td>
</tr>
<tr>
<td>Rated currents in terminals</td>
<td>18.2 A / 866 A</td>
</tr>
<tr>
<td>Rated currents in windings</td>
<td>10.5 A / 866 A</td>
</tr>
<tr>
<td>Rated number of turns in windings</td>
<td>1732 / 21</td>
</tr>
</tbody>
</table>

Main transformer parts relevant to the magnetic field calculation are (figure 1):

- windings
- low voltage (hereinafter referred to as "LV") terminals
- high voltage (hereinafter referred to as "HV") terminals + feeding cables
- core
- tank
- clamping system

Figure 1 - Main parts of oil-type distribution transformer

There are two groups of parts; primary sources and ferromagnetic materials.

In the group of primary sources are windings, LV terminals, HV terminals and feeding cables. Known currents flow through them. They are primary sources of the magnetic field (denoted by P in figure 1).

In the group of ferromagnetic materials are core, tank and clamping system. They were modeled by secondary sources (denoted by S in figure 1). The clamping system was ignored in the magnetic field calculation due to memory limitations and due to author’s evaluation that its influence on a magnetic field in the surrounding region of transformer is low. Core and tank were assumed to be linear isotropic homogeneous materials with two constant parameters; magnetic permeability - $\mu_r$ and electrical conductivity - $\kappa$ [6]. The values of $\mu_r$ and $\kappa$ are given in table II.

Table II - Relative permeability and electrical conductivity for core and tank

<table>
<thead>
<tr>
<th></th>
<th>relative permeability - $\mu_r$</th>
<th>electrical conductivity - $\kappa$ [MS/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>core</td>
<td>20000</td>
<td>0</td>
</tr>
<tr>
<td>tank</td>
<td>400</td>
<td>6.4</td>
</tr>
</tbody>
</table>

The core was made of classical grain oriented material - M5. Relative permeability of such material measured (0.07 T) and rated (1.63 T) magnetic induction is 20000 [6]. It’s worth mentioning that changes of $\mu_r$ in the 10000 to 50000 range do not affect the results significantly [6], [7]. Electrical conductivity of the core was assumed to be zero (no eddy currents) due to lamination of the core [6], [7], [10]. The values for the tank (construction steel) were obtained experimentally [6], [8].

3. METHOD FOR COMPUTING THE MAGNETIC FIELD

Today’s commonly used software based on differential approach (finite element method) is not appropriate tool for computing the magnetic field of the transformer. The points of interest are relatively far from the sources (windings, terminals). Domain of such problem is very large which results in huge
number of elements. Instead of differential approach, an integral approach was applied to the magnetic field computation. The method is based on equivalent sources in magnetic empty space ($\mu = \mu_0$). These equivalent sources replace the ferromagnetic material and they were called secondary sources. In a quasi-static analysis of a system of a known primary source and a linear ferromagnetic material (figure 2), the secondary sources are the following [6], [9]:

- $\vec{K}_m$ - surface current density ($A/m$) on the surface $S$ of ferromagnetic material
- $\sigma_v$ - surface charge density ($C/m^2$) on the surface $S$ of ferromagnetic material
- $\vec{J}_v$ - eddy current density ($A/m^2$) inside the volume $V$ of ferromagnetic material

![Figure 2 - Equivalent sources in a quasi-static model of linear ferromagnetic material](image)

All time-dependent variables were assumed to be sinusoidal functions of time. Analysis was made in phasor domain which was denoted by a point above the variable. $\vec{K}_m$ and $\vec{J}_v$ are vectors, while $\sigma_v$ is a scalar.

These sources generate the following magnetic field (Biot-Savart's law):

\[
\hat{H}_o(\hat{r}) = \frac{1}{4\pi} \int_{V_o} \vec{J}_o(\hat{r}') \times \frac{\hat{R}}{R^3} dV'
\] (1)

\[
\hat{H}_s(\hat{r}) = \frac{1}{4\pi} \int_{S} \vec{K}_m(\hat{r}') \times \frac{\hat{R}}{R^3} dS'
\] (2)

\[
\hat{H}_v(\hat{r}) = \frac{\mu_v(\hat{r})}{4\pi} \int_{V} \vec{J}_v(\hat{r}') \times \frac{\hat{R}}{R^3} dV'
\] (3)

where $R$ is the distance between the observation and integration points:

\[
\hat{R} = \hat{r} - \hat{r}'
\] (4)

Magnetic field at any point $\hat{r}$ of the space is

\[
\hat{H}(\hat{r}) = \hat{H}_o(\hat{r}) + \hat{H}_s(\hat{r}) + \hat{H}_v(\hat{r})
\] (5)

Magnetic induction is (space is considered "empty")

\[
\hat{B}(\hat{r}) = \mu_0 \cdot \hat{H}(\hat{r})
\] (6)
4. LINEAR SYSTEM OF INTEGRAL EQUATIONS

In order to find the magnetic induction, all sources must be known. The primary sources are known and to find the secondary ones, integral equations are written as follows [6], [9]:

\[
\dot{\mathbf{J}}_v(\vec{r}) + \frac{j_0\kappa\mu_0\mu_r}{4\pi} \int_V \dot{\mathbf{J}}_v(\vec{r}') \frac{dV'}{R} + \frac{j_0\kappa\mu_0}{4\pi} \int_S \dot{\mathbf{K}}_m(\vec{r}') \frac{dS'}{R} - \frac{\kappa}{4\pi\varepsilon_0} \int_S \sigma_v(\vec{r}') \frac{\vec{R}}{R^3} dS' = -j_0k\dot{\mathcal{A}}_0(\vec{r})
\]

(7)

\[
\dot{\mathbf{K}}_m(\vec{r}) - \frac{\lambda_m \cdot \mu_r}{2\pi} \int_V \left(\dot{\mathbf{J}}_v(\vec{r}') \times \frac{\vec{R}}{R^3}\right) \times \vec{n}(\vec{r}) dV' - \frac{\lambda_m}{2\pi} \int_S \left(\dot{\mathbf{K}}_m(\vec{r}') \times \frac{\vec{R}}{R^3}\right) \times \vec{n}(\vec{r}) dS' = 2 \cdot \lambda_m \cdot \dot{\mathcal{H}}_0(\vec{r}) \times \vec{n}(\vec{r})
\]

(8)

\[
\dot{\sigma}_v(\vec{r}) + \frac{j_0\mu_0\mu_r}{2\pi} \int_V \dot{\mathbf{J}}_v(\vec{r}') \frac{\vec{n}(\vec{r}')}{R} dV' + \frac{j_0\mu_0\varepsilon_0}{2\pi} \int_S \dot{\mathbf{K}}_m(\vec{r}') \cdot \frac{\vec{n}(\vec{r}')}{R} dS' - \frac{1}{2\pi} \int_S \sigma_v(\vec{r}') \frac{\vec{R}}{R^3} \cdot \vec{n}(\vec{r}') dS' = -2j_0\varepsilon_0 \dot{\mathcal{A}}_0(\vec{r}) \cdot \vec{n}(\vec{r})
\]

(9)

where
\[
\omega = 2 \cdot \pi \cdot f = 2 \pi \cdot 50 = 314.16 \text{ s}^{-1}
\]

\[
\mu_0 = 4 \cdot \pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}
\]

\[
\varepsilon_0 = 8.854 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}
\]

\[
\lambda_m = \frac{\mu_r - 1}{\mu_r + 1}
\]

\[
\dot{\mathcal{A}}_0(\vec{r}) = \frac{\mu_0}{4\pi} \int_{V_0} \dot{\mathbf{J}}_v(\vec{r}') \frac{dV'}{R}
\]

(10) (11) (12) (13) (14)

Equations (7), (8) and (9) can be written as a linear system in matrix form [6]:

\[
\mathbf{A} \cdot \mathbf{x} = \mathbf{b}
\]

(15)

where

\[
\begin{array}{c|c}
\mathbf{A} & \text{Matrix of the system} \\
\mathbf{x} & \text{Vector of unknown secondary sources} \\
\mathbf{b} & \text{Vector of primary sources} \\
\end{array}
\]

Dimensions

\[
\begin{array}{ccc}
3(N_V + N_S) & \times & 3(N_V + N_S) \\
3(N_V + N_S) & \times & 1 \\
3(N_V + N_S) & \times & 1 \\
\end{array}
\]

The ferromagnetic material was divided into \(N_V\) volume elements and \(N_S\) surface elements. Secondary sources were assumed to be constant all over the elementary volume and surface.
5. MODELS OF TRANSFORMER FOR MAGNETIC FIELD CALCULATION

Three transformer models for magnetic field calculation are shown in figures 4 - 6. Model 1 (figure 4) is the simplest and consists only of primary sources (windings, terminals). Model 2 (figure 5) additionally takes into account the transformer core. Model 3 (figure 6) is the most complex and consists of the primary sources, core and the tank.

Division of ferromagnetic materials (core and tank) depends on computer memory size. In this paper computations were made on a computer with 1.8 GHz CPU and 3.3 GB RAM. As stated before, eddy currents in the core were disregarded, and the core was divided only on surface elements (1892 elements). The tank was modeled with six plates. Due to memory limitations it was divided very rough (756 surface and 294 volume elements). Rectangle and cuboid were chosen as the elementary surface and volume elements, respectively.
6. COMPARISON BETWEEN CALCULATIONS AND MEASUREMENTS

Magnetic induction was measured and computed on 630 kVA transformer defined in table I. The transformer was energised from HV side and LV side was in short circuit (figure 3). Rated currents flew through windings and terminals. According to figures 3 and 7, the origin of coordinate system was set at the top of the tank cover (hereinafter referred to as “origin of transformer”). The magnetic induction was measured and computed in three planes (figure 7). Figures 8 - 10 show the comparison between measured and computed rms values of the magnetic induction at 50 Hz.

Figure 7 - Planes of measurements and calculations

Figure 8 - Comparison between calculations and measurements in XY plane (z=-0.355 m)

Figure 9 - Comparison between calculations and measurements in XZ plane (y=0 m)

Figure 10 - Comparison between calculations and measurements in YZ plane (x=0 m)

Maximum deviations between the calculations and measurements are given in table III.

<table>
<thead>
<tr>
<th></th>
<th>model 1 (primary sources)</th>
<th>model 2 (primary sources+core)</th>
<th>model 3 (primary sources+core+tank)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 μT</td>
<td>50 %</td>
<td>17 %</td>
<td>-</td>
</tr>
<tr>
<td>40 μT</td>
<td>100 %</td>
<td>56 %</td>
<td>18</td>
</tr>
</tbody>
</table>
Rather than maximum deviations, more important information is how far is the most distant point of the curves. The distances between the most distant points of the curves and the origin of transformer are given in table IV.

**Table IV – Distances between the most distant points of the curves and the origin of 630 kVA oil-type transformer**

<table>
<thead>
<tr>
<th></th>
<th>XY plane</th>
<th>XZ plane</th>
<th>YZ plane</th>
<th>Max. distance</th>
<th>Deviation between computed and measured max. distances</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 μT measured</td>
<td>2.66 m</td>
<td>2.46 m</td>
<td>2.67 m</td>
<td>2.67 m</td>
<td></td>
</tr>
<tr>
<td>computed (model 1)</td>
<td>3.20 m</td>
<td>3.22 m</td>
<td>2.90 m</td>
<td>3.22 m</td>
<td>20.6 %</td>
</tr>
<tr>
<td>computed (model 2)</td>
<td>2.56 m</td>
<td>2.51 m</td>
<td>2.58 m</td>
<td>2.58 m</td>
<td>-3.37 %</td>
</tr>
<tr>
<td>40 μT measured</td>
<td>0.83 m</td>
<td>0.90 m</td>
<td>0.79 m</td>
<td>0.90 m</td>
<td></td>
</tr>
<tr>
<td>computed (model 1)</td>
<td>1.31 m</td>
<td>1.37 m</td>
<td>0.96 m</td>
<td>1.37 m</td>
<td>52.2 %</td>
</tr>
<tr>
<td>computed (model 2)</td>
<td>1.08 m</td>
<td>1.14 m</td>
<td>0.84 m</td>
<td>1.14 m</td>
<td>26.7 %</td>
</tr>
<tr>
<td>computed (model 3)</td>
<td>0.90 m</td>
<td>0.94 m</td>
<td>0.87 m</td>
<td>0.94 m</td>
<td>4.44 %</td>
</tr>
</tbody>
</table>

Despite of high deviations for model 1, it can be useful when finding the most distant point of the magnetic induction curve. Computation on model 1 showed the most distant points of 1 μT and 40 μT curves to be 3.22 m and 1.37 m far from the origin of transformer, respectively. Maximum measured distances were 2.67 m and 0.90 m. That gives 21 % and 52 % deviation between measurements and computations. Furthermore, the analysis showed the influence of the terminals to be low. The computation on simplified model 1 (with windings only) resulted in distances 3.19 m and 1.33 m, which are almost the same as for the model with both windings and terminals. Curves of magnetic induction computed on model 1 are always farther from transformer than measured curves (figures 8-10). Therefore we are always on the safe side. The model 1 is recommended for quick magnetic field computation especially when the points of interest are several meters from the transformer.

Model 2 gives much better results than model 1, but deviation is still high for the points in the close proximity of transformer. This model is also used for magnetic field computation of dry-type transformer [7] and cast-resin transformer [10].

To compute the magnetic field in the close proximity of oil-type distribution transformer, model 3 is needed. This model is memory demanding and time consuming. Also, fine division of ferromagnetic material is necessary. The induced eddy currents distribute in a very thin layer on its surface. Therefore the tank should be divided into layers which significantly increases the number of elements. In mentioned hardware configuration (3.3 GB RAM), number of elements was to small to compute the magnetic field adequately in all regions, except in regions where the field is generated dominantly by inner sources (windings).

### 7. MAGNETIC FIELD EVALUATION OF DISTRIBUTION TRANSFORMERS

Simplified model 1 was used to compute the magnetic field of oil-type distribution transformers in the range 50-2500 kVA. The distances between the most distant point of 1 μT and 40 μT curve and the origin of distribution transformers are shown in figure 11. Measured values are also shown for 630 and 1000 kVA transformers. All transformers were SI 24.

*Figure 11 – Maximum distances between curves of magnetic induction and the origin of oil-type distribution transformers*
As stated before, such computed distances should be higher than measured ones, especially for 40 μT curve. In relation to exposure to the magnetic fields, even if these computed values are taken into account, the 40 μT limit should not be the problem due to low distances from the transformer. But, 1 μT limit might be the problem.

In this analysis the only source of the magnetic field was assumed to be the transformer itself. But when the transformer is in operation (in network), dominant source of the magnetic field could be LV conductors. The magnetic field can be significantly decreased by using LV twisted cables. If it is not possible, the individual LV phase conductors should be as close as it’s possible. LV bus-bars are not recommended. If they are used, the magnetic field can be decreased by shielding of bus-bars.

8. CONCLUSION

Although the satisfying results for magnetic induction are achievable for oil-type distribution transformer, the lack of computer memory is still the major limitation.

The advantage of proposed method based on integral approach is possibility to compute the magnetic field very simply and quickly, especially when the computed values are several meters from the transformer.

9. REFERENCES

[5] EN 50464-1 Three-phase oil-immersed distribution transformers 50 Hz, from 50 kVA to 2500 kVA with highest voltage for equipment not exceeding 36 kV – Part 1: General requirements, April 2007.