STOCHASTIC MODELING OF PUMP-STOREAGE HYDROELECTRIC POWER PLANTS, Part I

Lajos Józsa, Damir Šljivac, Danijel Topić

Two parts of this paper represent a contribution to the implementation of a pump-storage hydroelectric power plant stochastic model into a power plant system reliability model. After analysis of operation modes of such type of power plants, Part I gives an overview referring to the determination of the probability distribution of variable production when natural hydraulic inflow into the upper storage is exclusively used, the probability distribution of variable production from a pumped-storage drive, as well as the probability distribution of necessary energy of the plant with natural hydraulic inflow. The inflow is thereby treated as a random variable, and stochastic modeling relies on the well-known method of constant and variable energy production. The developed model is suitable for development of an additional determination criterion when it comes to making studies related to planning strategy of production capacities in the power system. The stochastic model presented in the paper is illustrated by a simple numerical example.

Key words: density function, distribution function, method of constant and variable energy production, probability function, reliability, stochastic inflow

1 Introduction

For the purpose of accumulating a surplus of hydropower during off-peak periods, a pump-storage hydroelectric plants are built, which have balancing impact on the load curve. Within the pump-storage hydroelectric plants by means of low-cost off-peak electric power water is pumped from the lower storage into the upper storage, which is then in other periods transformed into peak energy of great value. Two types of pump-storage hydroelectric plants may be distinguished; i.e. the ones with and without natural hydraulic inflow into the upper storage. In both cases it is assumed that pumps on the suction side have a sufficient amount of water available for pumping at any time. The developed stochastic model can be used for reliability estimation of the electric power generating system as an auxiliary criterion by making studies related to planning strategies of generating capacities in the power system.

Prior to presentation of the insertion of a pump-storage hydroelectric plant's stochastic model into the stochastic modelling of overall system of hydro and thermal power plants it is necessary to briefly analyse the operation modes of such plants [2, 3, 4].

2 Analysis of a pump-storage hydroelectric plant operation mode

Pump-storage hydroelectric power plants must meet the same requirements with respect to operation mode strategy as hydroelectric storage plants. Indeed, they need to produce more variable energy in the shortest time possible with maximum capacity. The method of constant and variable energy production described in [5] enables the determination of the possible, variable and constant production from hydroelectric storage plants in the week observed. In case of hydroelectric storage plants, consideration of inflow as a random variable, as described in [3], requires determination of the probability distribution of possible, variable and constant production. Dependencies used for that purpose according to the method of constant and variable energy production are shown in the Appendix, and they are also applied in the case of pump-storage hydroelectric power plants. Assuming for the purpose of simplicity that after six working days there follows one non-working day when variable energy is not needed, Fig. 1 shows the operation modes diagram of a pump-storage hydroelectric plant without (Fig. a) and with (Fig. b) natural inflow.

Symbols given in Fig. 1 mean the following: \( Q_t \) – maximum turbine flow-through (design size), (m\(^3\)/s); \( P_{\text{max}} \) – maximum capacity of a pump-storage plant in turbine drive, (MW); \( Q \) – natural inflow, (m\(^3\)/s); \( P \) – capacity a plant would have if it used water corresponding to natural inflow, (MW); \( Q_{\text{max}} \) – maximum pump flow-through in the observed week, (m\(^3\)/s); \( P_{\text{pmax}} \) – pump capacity
Weekly amount of energy a plant without an inflow can produce for $r$ working days with the maximum capacity of turbine drive during the time $t_r$ is:

$$W_{ir, \text{max}} = P_{h, \text{max}}r t_r \times 10^{-3}, \text{ GWh}$$  \hspace{1cm} (3)$$

Energy required for the pump drive mode should be provided for by other power plants in the system. Off-peak load periods are favorable for this (see Fig. D-1).

$$T_m = T - r_t = 24 \cdot (r + n) - r_t.$$  \hspace{1cm} (4)$$

The dependence of the maximum pump flow-through $Q_{p, \text{max}}$ and the maximum possible duration of using maximum power in turbine drive can be drawn from the condition that in the case of a pump-storage plant without natural inflow the quantity of the pumped water must be equal to the quantity used in turbine drive:

$$Q_{t, \text{max}} = 24 \cdot (r + n) = t'_v.$$  \hspace{1cm} (5)$$

Equations (2) to (5) enable calculation of the weekly amount of energy by which water required for production of $W_{ir, \text{max}}$ is pumped into the upper storage:

$$W_{pr, \text{max}} = P_{hp, \text{max}}T_m \times 10^{-3} = \frac{P_{hp, \text{max}}r t_r \times 10^{-3}}{\eta_t \cdot \eta_p \cdot \alpha}$$  \hspace{1cm} (6)$$

In equations (1) and (2): $H_t$ – net head (m); $H_g$ – gross head (m); $\alpha$ – net-gross head ratio ($\alpha < 1$); $\eta_t$ – efficiency rate of the plant in turbine drive; $\eta_p$ – efficiency rate of the plant in pump drive.
If there is a natural inflow into the upper storage, it will be used for energy production. How long a power plant with maximum power on working days can produce variable energy from a natural inflow is determined on the basis of the condition that the quantity of water inflowing during \(24(\tau + n)=168\) hours with flow-through \(Q\) must be equal to the quantity of water that can be used for energy production on working days at time \(t_\alpha\) at maximum flow-through (see Fig. 1-b):

\[
t_\alpha = \frac{Q \cdot 24(\tau + n)}{Q_A \cdot r}, \text{ h} \tag{7}
\]

In this case energy production on a working day during \(t_\alpha\) hours is obtained partially from the natural inflow and partially from a pumped-storage drive. For \(t_\alpha \leq t_\alpha\), it is hence necessary to predict water pumping into the upper storage, i.e. pumping of those quantities which together with the natural inflow enable usage of maximum power during \(t_\alpha\) hours in a working day. During time interval \(t_\alpha\), energy is produced by means of water which inflows naturally, whereas in time interval \(t_\alpha<t_\alpha\), the pumped water is used.

From Fig. 1-b it can be seen that for variable energy from a natural inflow the following equation holds:

\[
W_{na} = P_{h,max} r t_{\alpha} \times 10^{-3}, \text{ GWh} \tag{8}
\]

and for variable energy from a pumped-storage operation:

\[
W_{ps} = P_{h,max} r (t_\alpha - t_\alpha) \times 10^{-3} = W_{st,\max} - W_{na}, \text{ GWh} \tag{9}
\]

Pump flow-through is determined similarly to the case of a plant without a natural inflow:

\[
Q_p = Q_A \cdot \frac{r(t_\alpha - t_\alpha)}{24(\tau + n) - r t_\alpha} \tag{10}
\]

Pump capacity necessary for that flow-through is:

\[
P_p = 9.81 \cdot Q_p \cdot H_B \frac{1}{n_p} \times 10^{-3}, \text{ MW} \tag{11}
\]

By means of equations (1), (4), (7), (10) and (11) the dependence between natural inflow \(Q\) and pump capacity \(P_p\) is established:

\[
P_p = \frac{P_{h,max} \cdot r t_{\alpha}}{n_p \cdot T_{m,\alpha}} = \frac{P_{h,max}}{n_p \cdot n_{\alpha} \cdot T_{m,\alpha}} \cdot 24(\tau + n) \cdot Q, \text{ MW} \tag{12-a}
\]

Under the assumption of a constant gross head and the efficiency rate the following linear function is obtained:

\[
P_p = C_1 - C_2 Q, \text{ MW} \tag{12-b}
\]

Shown in Fig. 2:

If there are \(i<n\) pumps in operation, there is a family of curves which can also be seen in Fig. 2 for the case with four pumps and whose constant section can be explained in the following way: If the quantities of a natural inflow are less than \(Q(i)\), a pumping capacity will be necessary according to function \(P_p = f(Q)\) in order to provide for the requested production \(W_{\text{max}}\) from water pumped additionally at time \(T_m\). However, for \(Q \leq Q(i)\), for every \(i\) only capacity \(P_{sp,i}\) is available.

The necessary pumping power is obtained on the basis of expressions (4), (10) and (11):

\[
W_{ps} = P_p\cdot T_m \times 10^{-3} = 9.81 \cdot Q_A \cdot H_B \frac{1}{n_p} \times 10^{-6} r (t_\alpha - t_\alpha) = \tag{13}
\]

\[
= \frac{P_{h,max} r}{n_p \cdot n_{\alpha} \cdot T_{m,\alpha}} \cdot (W_{st,\max} - W_{na}), \text{ GWh}
\]

Figure 3 Modification of the load duration curve with a pump-storage hydroelectric power plant with natural inflow

1 – Original duration curve, 2 – Modified duration curve

Slika 3. Modifikacija krivulje trajanja opterećenja kod crpno-akumulacijske hidroelektrane s prirodnim dotokom

1 – originalna krivulja trajanja, 2 – Modificirana krivulja trajanja

\[
W_{ps} = P_p\cdot T_m \times 10^{-3} = 9.81 \cdot Q_A \cdot H_B \frac{1}{n_p} \times 10^{-6} r (t_\alpha - t_\alpha) = \tag{13}
\]

\[
= \frac{P_{h,max} r}{n_p \cdot n_{\alpha} \cdot T_{m,\alpha}} \cdot (W_{st,\max} - W_{na}), \text{ GWh}
\]
From the aforementioned follows the way a pump-storage plant with natural inflow – by using of maximum capacity on working days during $t_{w}^\ast < t_{w}$ hours, as well as with a 100% reliable pump and turbine drive – modifies the load duration curve (Fig. 3).

Energy production can naturally be even greater than $W_{va,max}$, if it is caused by such natural inflow into the upper storage which enables usage of maximum capacity on working days during $t_{w}^\ast < t_{w} \leq t_{w}^\ast$. In such a case pump drive is superfluous, because there is enough water from the natural inflow. Then, a pump-storage power plant becomes a normal hydroelectric storage plant which produces additional variable energy, and for higher inflows even constant energy.

3 Determining probability distributions of variable production and necessary pumping energy of a pump-storage hydroelectric plant with natural inflow

A stochastic character of a possibly existing natural hydraulic inflow mostly determines modeling of pump-storage hydroelectric power plants with respect to reliability calculations. For $t_{w}^\ast < t_{w}$, the total production of such power plants is certainly equal to the deterministic value $W_{va,max}$ – whether that energy is produced by exclusive pump-storage operation (without natural inflow) or by partial utilization of the natural inflow with additional production from the pump-storage operation. For pumping energy that should be probability distribution can be established conditioned by stochastic natural inflows. Initial base here is also the natural inflow probability distribution described in [3]. At first from this distribution, in the way shown for storage hydro plants also in [3], as well as by means of dependence (D-5) and equality:

$$W_{va} = W_{va,max} ; \quad 0 \leq W_{va} \leq W_{va,max}$$

(14)

can be created the probability density function $f(W_{va})$ of variable production of a pump-storage hydro plant using exclusively the natural inflow.

The probability of appearance of an inflow from interval $(Q'; Q' + \Delta Q)$ equals to the probability that the corresponding variable production for exclusive use of natural inflow, determined by (D-5) and (14), lies in interval $(W_{va}' , W_{va}' + \Delta W_{va}')$:

$$\operatorname{Pr}(Q' < Q \leq Q' + \Delta Q) = f(Q)\Delta Q = \operatorname{Pr}(W_{va}' < W_{va} \leq W_{va}' + \Delta W_{va}) = f(W_{va})\Delta W_{va}.$$  

(15)

The required density function follows from this:

$$f(W_{va}) = \frac{f(Q)\Delta Q}{\Delta W_{va}} = \frac{\operatorname{Pr}(Q' < Q \leq Q' + \Delta Q)}{\Delta W_{va}}.$$  

(16)

In Fig. 4, containing a graph-analytical determination of function $f(W_{va})$, it can be seen that generally two cases might occur, i.e. for $Q_{max} \leq Q_{1}$ and $Q_{max} > Q_{1}$. Here is $Q_{max}$ the maximum inflow in the observed week and $Q_{1}$ the inflow value which during $t_{w}^\ast$ hours on working days in the observed week results in production $W_{va,max}$. $Q_{1}$ is calculated from

![Figure 4](image-url)  

*Figure 4* Determining the variable production density function of a pump-storage hydroelectric plant with exclusive usage of natural inflow  
*Slika 4.* Određivanje funkcije gustoće varijabilne proizvodnje crpno-akumulacijske hidroelektrane sa isključivo prirodnim dotokom
equation (D-5), (D-6) and (3) with \( W_n = W_{\text{vac}} \):

\[
Q_t = Q_A \cdot \frac{r_{vu}}{24(r+n)} \text{ m}^3/\text{s} \tag{17}
\]

In case 2 \((Q_{\text{vac}} > Q_t)\), the density function is limited at \( W_{\text{vac}} \), causing thereby a discrete jump given by the following equation:

\[
\Pr(W_{\text{vu},\text{max}}) = \Pr(W_{\text{vu},\text{max}1}) = \int_{W_{\text{vu},\text{min}1}}^{W_{\text{vu},\text{max}1}} f_2(W_{\text{vu}})dW_{\text{vu}} = \frac{Q_{\text{max}}}{Q_t} = \int f(Q)dQ.
\tag{18}
\]

The obtained probability density function \( f(W_{\text{vu}}) \) satisfies the condition:

\[
\int_{-\infty}^{\infty} f(W_{\text{vu}})dW_{\text{vu}} = \int_{W_{\text{vu},\text{min}1}}^{W_{\text{vu},\text{min}2}} f_2(W_{\text{vu}})dW_{\text{vu}} = f_2(W_{\text{vu}})dW_{\text{vu}} = 1.
\tag{19}
\]

and it has the expected value (for case 1 and 2):

\[
E_1(W_{\text{vu}}) = \int_{-\infty}^{\infty} W_{\text{vu}}f_1(W_{\text{vu}})dW_{\text{vu}} = \int_{W_{\text{vu},\text{min}1}}^{W_{\text{vu},\text{min}2}} W_{\text{vu}}f_1(W_{\text{vu}})dW_{\text{vu}} , \text{ GWh}
\tag{20-a}
\]

\[
E_2(W_{\text{vu}}) = \int_{W_{\text{vu},\text{min}1}}^{W_{\text{vu},\text{min}2}} W_{\text{vu}}f_2(W_{\text{vu}})dW_{\text{vu}} = \int_{W_{\text{vu},\text{min}1}}^{W_{\text{vu},\text{min}2}} W_{\text{vu}}f_2(W_{\text{vu}})dW_{\text{vu}} + \int_{W_{\text{vu},\text{max}2}}^{W_{\text{vu},\text{max}1}} W_{\text{vu}}f_2(W_{\text{vu}})dW_{\text{vu}} = \tag{20-b}
\]

The just determined probability density function of variable energy with exclusive usage of natural inflow, together with dependence (9), is now available for determination of variable energy density function \( f(W_{\text{ps}}) \) produced by the pump-storage cycle. Calculation of function \( f(W_{\text{ps}}) \) is based upon the following probability equality:

\[
\begin{align*}
\Pr(W_{\text{vu}} < W_{\text{ps}} & \leq W_{\text{ps}} + \Delta W_{\text{ps}}) = f(W_{\text{ps}})\Delta W_{\text{ps}} \\
\Pr(W_{\text{ps}} < W_{\text{ps}} & \leq W_{\text{ps}} + \Delta W_{\text{ps}}) = f(W_{\text{ps}})\Delta W_{\text{ps}}. \tag{21}
\end{align*}
\]

\( W_{\text{ps}} \) is here energy which on the basis of (9) corresponds to \( W_{\text{vu}} \) and which is obtained from the water pumped into the upper storage.

\[
\begin{align*}
\Pr(W_{\text{vu}} < W_{\text{ps}} & \leq W_{\text{ps}} + \Delta W_{\text{ps}}) = \frac{\Delta W_{\text{ps}}}{f(W_{\text{ps}})} \Rightarrow \frac{\Pr(W_{\text{vu}} < W_{\text{ps}} \leq W_{\text{ps}} + \Delta W_{\text{ps}})}{\Delta W_{\text{ps}}}, \tag{22}
\end{align*}
\]

The graph-analytical determination of the function \( f(W_{\text{ps}}) \) for both cases 1 and 2 is shown in Fig. 5.

In equation (9) which defines the dependence of \( W_{\text{ps}} \) and \( W_{\text{vu},\text{max}} \), occurs as a constant given by expression (3). The jump in \( f(W_{\text{ps}}) \) at \( W_{\text{vu}} = 0 \) for case 2 is determined by probability:

\[
\begin{align*}
Pr(W_{\text{vu},\text{max}1} & = W_{\text{vu},\text{max}2}) = \int f(W_{\text{vu}})dW_{\text{vu}}.
\end{align*}
\]

Figure 5 Determining the probability density function of the variable production from the pump-storage operation

Slika 5. Određivanje funkcije gustoće vjerojatnosti varijabilne proizvodnje kod crpno-akumulacijskog pogona
\[
\Pr(W_{ps} \leq 0) = \int_{W_{ps, min,2}}^{W_{ps, max,2}} f_2(W_{ps}) \, dW_{ps} = \int_{W_{vt, max}}^{W_{vt, max}} f_2(W_{va}) \, dW_{va} = \Pr(Q > Q_t) = \int_{\Omega}^{\max} f(Q) \, dQ.
\]

The following condition is also satisfied:
\[
\int_{-\infty}^{\infty} f(W_{ps}) \, dW_{ps} = \int_{W_{ps, min,1}}^{W_{ps, max,1}} f_1(W_{ps}) \, dW_{ps} = \int_{W_{ps, min,2}}^{W_{ps, max,2}} f_2(W_{ps}) \, dW_{ps} = 1. \tag{24}
\]

The expected value (for both cases) is defined in a well-known way:
\[
E_1(W_{ps}) = \int_{-\infty}^{\infty} W_{ps} \cdot f_1(W_{ps}) \, dW_{ps} = \int_{W_{ps, min,1}}^{W_{ps, max,1}} W_{ps} \cdot f_1(W_{ps}) \, dW_{ps}, \text{ GWh} \tag{25-a}
\]
\[
E_2(W_{ps}) = \int_{-\infty}^{\infty} W_{ps} \cdot f_2(W_{ps}) \, dW_{ps} = \int_{W_{ps, min,2}}^{W_{ps, max,2}} W_{ps} \cdot f_2(W_{ps}) \, dW_{ps}, \text{ GWh} \tag{25-b}
\]

The sum of expected values of variable production with an exclusive usage of natural inflow and with pumped-storage drive gives maximum variable production of a pump-storage plant in turbine drive with maximum capacity on working days during the observed week:
\[
W_{vt, max} = E(W_{va}) + E(W_{ps}). \tag{26}
\]

The function \( f(W_{va}) \) will also be needed for determining the probability distribution of the required pumping energy of the plant with natural inflow. Hence the following equations can be written by applying similar logic as above:
\[
\Pr(W_{va} < W_{va} \leq W_{va} + \Delta W_{va}) = f(W_{va}) \, \Delta W_{va} = \Pr(W_{ps} < W_{ps} \leq W_{ps} + \Delta W_{ps}) = f(W_{ps}) \, \Delta W_{ps} \tag{27}
\]
\[
f(W_{ps}) = \frac{f(W_{va}) \, \Delta W_{va}}{\Delta W_{ps}} \tag{28}
\]
\[
\Pr(W_{ps} = 0) = \int_{W_{ps, min,2}}^{W_{ps, max,2}} f_2(W_{ps}) \, dW_{ps} = \Pr(Q > Q_t) \tag{29}
\]
\[
E_1(W_{ps}) = \int_{-\infty}^{\infty} W_{ps} \cdot f_1(W_{ps}) \, dW_{ps} = \int_{W_{ps, min,1}}^{W_{ps, max,1}} W_{ps} \cdot f_1(W_{ps}) \, dW_{ps}, \text{ GWh} \tag{31-a}
\]
\[
E_2(W_{ps}) = \int_{-\infty}^{\infty} W_{ps} \cdot f_2(W_{ps}) \, dW_{ps} = \int_{W_{ps, min,2}}^{W_{ps, max,2}} W_{ps} \cdot f_2(W_{ps}) \, dW_{ps}, \text{ GWh} \tag{31-b}
\]

A graph-analytical representation can be seen in Fig. 6.

\[\text{Figure 6 Determining the probability density function of the required pumping energy for pump-storage plant with natural inflow} \]

\[\text{Slika 6. Određivanje funkcije gustoće vjerovatnosti crpljenja potrebne energije kod crpno-akumulacijskog postrojenja s prirodnim dotokom}\]
Distribution function $F(W_{pm})$, i.e. its complement, the probability function $F^*(W_{pm})$ of required pumping energy in case when natural inflow exists, is especially important for the forthcoming modeling of pump-storage power plants within the scope of a complete power plant system model. In relation to this, the following equations may be written:

$$
F(W_{pm}) = 0
$$

$$
F_{1}(W_{pm}) = \int_{w_{pm, min}}^{w_{pm, max,1}} f_{1}(W_{pm})dW_{pm} = \int_{w_{pm, min,1}}^{w_{pm, max,1}} f_{1}(W_{pm})dW_{pm};
$$

$$
F_{1}(W_{pm}) = 1
$$

$$
F^*(W_{pm}) = W_{pm} < W_{pm, min,1}
$$

$$
F^*(W_{pm}) = W_{pm, min,1} \leq W_{pm} < W_{pm, max,1}
$$

$$
F^*(W_{pm}) = W_{pm} \geq W_{pm, max,1}
$$

$$
F^*(W_{pm}) = W_{pm} < W_{pm, min,1}
$$

$$
F^*(W_{pm}) = W_{pm, min,1} \leq W_{pm} < W_{pm, max,1}
$$

$$
F^*(W_{pm}) = W_{pm} \geq W_{pm, max,1}
$$

$$
F_{2}(W_{pm}) = 0
$$

$$
F_{2}(W_{pm}) = 1
$$

$$
F_{2}^*(W_{pm}) = 1;
$$

$$
F_{2}^*(W_{pm}) = W_{pm} < 0
$$

$$
F_{2}^*(W_{pm}) = 0 \leq W_{pm} < W_{pm, max,2}
$$

$$
F_{2}^*(W_{pm}) = W_{pm} \geq W_{pm, max,2}
$$

The above functions are shown in Fig. 7.

The right hand-side marginal value of the probability function at $W_{pm}=0$ that in both cases determines the probability according to which a pump-storage power plant with natural inflow should pump water into the upper storage in the observed week:

$$
F^*(W_{pm}) = \Pr (W_{pm} > 0) = \int_{W_{pm, max,1}}^{W_{pm, max,2}} f_{1}(W_{pm})dW_{pm} = 1
$$

$$
F^*(W_{pm}) = \Pr (W_{pm} > 0) = \int_{W_{pm, max,2}}^{Q_{max}} f_{2}(W_{pm})dW_{pm} = \int_{Q_{min}}^{Q_{max}} f(Q)dQ. \tag{36-a}
$$

Finally, it should be mentioned that - as described in [3] and mentioned in the Appendix – considerations in this Section refer to the case when the relative useful volume of the storage is equal to or greater than the relative required volume for the maximum variable energy production ($a_t \geq a_{\max}$). Thereby it is assumed that $t_v \leq t_v$, which usually holds in practice.

As already mentioned, in pump-storage plants with natural inflow pump drive will be maintained as long as the inflowing water quantities and quantities of the water pumped into the upper storage together result in production $W_{t,v}$. Although the inflow is subject to the probability distribution, production is determined and it equals $W_{t,v}$. For higher inflows ($Q > Q$), as also mentioned, a pump-
storage power plant becomes a common hydroelectric storage plant. Dependences from [3] that refer to determination of the constant energy probability distribution can thus be applied in this case as well.

The following simple example provides a numerical proof for the above considerations.

4 Example

Determining the needed probability distributions of the pump-storage plant with natural inflow

Input data required for calculation are:

- The probability density function of the inflow is shown in the third quadrant of the coordinate system in Fig. 9.
- Data for the pump-storage plant are as follows:

  - Maximum turbine flow-through (designed size): \(Q_t = 107,380,189\) m³/s; gross (net) head (for the purpose of simplicity, it is assumed that gross and net head are equal, \(\alpha = 1\)); \(H_t = H_e = 100\) m; efficiency rate of the plant in turbine drive: \(\eta_t = 0.95\); efficiency rate of the plant in pump drive: \(\eta_p = 0.8\); useful storage volume: \(V_t = 4\) hm³; number of equal generator units: \(4\); rated power of generator units in turbine drive: \(P_{t,\text{max}} = 25\) MW; maximum capacity of the plant in the turbine drive during the observed week: \(P_{t,\text{obs}} = 100\) MW; outage probability in turbine drive: \(Pr(A) = 0.1\).

- Load data are given by the load duration curve shown in Fig. 8, as well as by the following values:

  - Maximum duration of using variable power plant capacity (adopted value): \(r_{\text{obs}} = 36\) h; number of working and non-working days in the observed week: \(r = 6\), \(n = 1\); duration of using maximum capacity in turbine drive (adopted value): \(t_{\text{max}} = 5\) h.

Determination of the probability density function for variable generation \(f(W_{\text{va}})\) by using natural inflow only is shown in Fig. 9.

\[f(W_{\text{va}})/1\text{GWh} = \begin{cases} 0.1 & \text{for} \ W_{\text{va}} = 0 \text{GWh} \\ 0.2 & \text{for} \ W_{\text{va}} = 1 \text{GWh} \\ 0.3 & \text{for} \ W_{\text{va}} = 2 \text{GWh} \\ 0.4 & \text{for} \ W_{\text{va}} = 3 \text{GWh} \end{cases}\]

\[W_{\text{ps}} = 3 - W_{\text{va}}, \text{GWh}\]

Fig. 10 represents the determination of probability density function \(f(W_{\text{ps}})\) of energy provided by the pump-storage drive. The required dependence \(W_{\text{ps}} = f(W_{\text{va}})\) is obtained by (9):

\[W_{\text{ps}} = 3 - W_{\text{va}}, \text{GWh}\]
In order to determine the probability distribution of the required pumping energy (Fig. 11), dependence $W_{pm} = f(W_{va})$ is calculated from (13):

$$W_{pm} = 3.947368421 - 1.315789474 \cdot W_{va} \text{ GWh}$$

The sum of the expected values $E(W_{pm})=1.6 \text{ GWh}$ and $E(W_{va})=1.4 \text{ GWh}$, calculated according to (20) and (25), meets the condition (26).

The probability of pump drive commitment can be determined from the function $F(W_{pm})$ for $W_{pm}=0$ according to expression (33), therefore: $Pr(W_{pm}>0)=1$.

Relative required storage volume for maximum variable energy generation:

$$a_{max} = \frac{rt_{vh} \cdot 2 \cdot 24 - t_{vh}}{24 (r + n)} \cdot 10^{-3} \quad (D-2)$$

Here are: $t_{vh}$ (least daily duration of using variable capacity of the hydro power plant, given in hours), $r$ – number of working days, $n$ – number of non-working days in the observed period.

5 Appendix

Dodatak

The method of constant and variable energy for the case of deterministic approach for determining available amounts of water used for energy generation is described in [5] in more detail. In the stochastic approach to storage hydro plant and pump-storage hydro plant modeling the following dependencies of this method are used:

Weekly load duration curve:

It is approximated so that the loading during off-peak load is substituted by horizontal according to Fig. D-1, as a result of which the demanded energy, represented by the area under the curve, is divided into constant and variable.

In Fig. D-1 are: $t_{vh}$ – daily peak load duration, $r$ – number of working days in a week, $T_{m}$ – weekly off-peak load duration.

Relative useful storage volume:

$$a_{k} = \frac{V_{k} \cdot 10^{3}}{3.6 \cdot 24 \cdot Q_{A}} = \frac{W_{k}}{24 \cdot P_{h,max} \cdot 10^{3}} \quad (D-1)$$

There are: $V_{k}$ – useful storage volume in $\text{hm}^{3}$, $Q_{A}$ – designed size of hydroelectric storage plant in $\text{m}^{3}/\text{s}$, $W_{k}$ – energy value of useful storage volume in $\text{GWh}$, $P_{h,max}$ – maximum capacity of the storage plant in the observed period in $\text{MW}$.

Figure D-1 Partition of the demanded energy into variable and constant energy. Slika D-1. Podjela potrebne energije u varijabilnu i stalnu

Relative required storage volume for maximum variable energy generation:

$$W_{hm} = \frac{W_{h,max} \cdot Q}{Q_{A}} \quad (D-5)$$

$W_{hm}$ is the height of the reservoir, two scenarios are possible: $a_{k} \geq a_{max}$ and $a_{k} < a_{max}$, but only the first scenario will be considered in this article ($a_{k} \geq a_{max}$).

- Use of water by the storage plant:

Depending on inflow two cases are distinguished as well as shown in the diagram in Fig. D-2 representing the use of water in the storage hydro plant.

The first case is when the natural inflow is low (i.e. the hydro plant capacity $P_{h}$ corresponding the inflow is low), the entire amount of water can be used to generate only variable energy $W_{va}$ during the time $t_{va}$ on working days, which equals the possible generation $W_{va}$.

$$W_{va} = W_{hm}; \quad 0 \leq W_{hm} \leq W_{va} \quad (D-3)$$

$$W_{vh} = 0; \quad 0 \leq W_{hm} \leq W_{va} \quad (D-4)$$

The possible generation is calculated from the inflow $Q_{d}$ designed size $Q_{A}$ and maximum generation $W_{h,max}$:

$$W_{h,max} = \frac{W_{h,max} \cdot Q}{Q_{A}} \cdot 10^{-3} \quad (D-7)$$

The duration of using variable capacity of the plant $t_{va}$ is given by the following equation:
If there are higher amounts of inflow, constant energy must be generated to, where $t_{va}$ just equals $t_{vc}$. Those amounts of energy are determined from the following expressions:

$$t_{va} = \frac{Q}{Q_A} \cdot \frac{24( r + n )}{r} = \frac{P_h}{P_{h,\text{max}}} \cdot \frac{24( r + n )}{r}$$  \hspace{1cm} (D-8)

The limit value between these two cases is at the value of possible generation:

$$W_{h,\text{max}}' = \frac{W_{h,\text{max}} \cdot r_{t_{ch}}}{24( r + n )}.$$  \hspace{1cm} (D-11)

### References

Literatura


### Authors’ addresses

Adrese autora

Prof. dr. sc. Lajos Józsa
Elektrotehnički fakultet
Sveučilište J. J. Strossmayera u Osijeku
Kralja Trpimira 2b
31000 Osijek, Croatia
email: lajos.jozsa@etfos.hr

Prof. dr. sc. Damir Šljivac
Elektrotehnički fakultet
Sveučilište J. J. Strossmayera u Osijeku
Kralja Trpimira 2b
31000 Osijek, Croatia
e-mail: damir.sljivac@etfos.hr

Danijel Topić, dipl. ing. elektrotehnike
Elektrotehnički fakultet
Sveučilište J. J. Strossmayera u Osijeku
Kralja Trpimira 2b
31000 Osijek, Croatia
e-mail: danijel.topic@etfos.hr