THE CSR SHIP HULL GIRDER ULTIMATE STRENGTH CHECK PROCEDURE REVISITED

Summary

This technical note firstly reminds on the idealized elastic-plastic modeling of the ultimate strength of structural elements. Next it reviews the ultimate strength assessment procedures for the ship’s hull supported by the Common Structural Rules (CSR) and by the American Petroleum Institute (API) standards. In the continuation the study compares the idealized ultimate strength assessments according to the flow stress approach recommended by the API-579 standards and the minimal yield strength approach as it is adopted by the CSR. At the end, the two approaches are separately applied to recently build five tankers and three bulk carriers following the simplified iterative-incremental method for ultimate strength assessment of the ship hull girder implemented in the program MARS. The paper indicated the differences in participation of buckling and plastic yielding in the total ultimate bending strength of the hull girder. The conclusion is that there are significant potential reserves in the hull girder ultimate strength with respect to the various assessments based on the idealized computational procedure recommended by the CSR.

Key words: ship hull, common structural rules, API standards, ultimate strength

PREISPITIVANJE ZAJEDNIČKIH PROPISA ZA PROCJENU GRANIČNE ČVRSTOĆE BRODSKOG TRUPA

Sažetak

Ovaj tehnički prikaz najprije podsjeća na idealizirano elasto-plastično modeliranje granične čvrstoće strukturnih elemenata. Potom daje pregledne postupke za procjenu granične čvrstoće brodskog trupa podržane u zajedničkim propisima za konstrukciju Common Structural Rules (CSR) i one dane u standardima američke naftne industrije American Petroleum Institute (API). U nastavku se uspoređuju idealizirane procjene prema naprezanju tečenja preporučene od strane API-579 standarda i prema minimalnom naprezanju popuštanja prema CSR. Na kraju, dva su pristupa zasebno primijenjena u procjeni granične čvrstoće trupova pet nedavno izgrađenih tankera i tri broda za rasute terete primjenom preporučenog pojednostavljenog iterativno-inkrementalnog postupka i to provedbom sa programom MARS. U radu su pokazani i udjeli izvijanja i plastičnog popuštanja u ukupnoj graničnoj čvrstoći brodskog trupa. Zaključak je ovog ispitivanja da postoje značajne zalihe u graničnoj čvrstoći brodskog trupa obzirom na različite procjene koje daje idealizirani računski postupak preporučen zajedničkim propisima.

Ključne riječi: brodski trup, zajednička pravila za trup, API standardi, granična čvrstoća
1. Introduction

The elastic-perfectly-plastic idealization of material behavior in assessment of the ultimate strength is commonly used in practical structural engineering and engineering of materials. Subsequently, the CSR for tankers and bulk carriers (IACS, 2006) applies material behavior idealization in assessment of the plastic collapse of hull girder scantlings embedded as the load-end shortening curves in the iterative-incremental calculation procedure. However, the results in prediction of the elastic-plastic collapse depend on the true material properties and may significantly differ for various idealization methods. Therefore the study considers the current CSR with respect to the practice of elastic-plastic modeling of steel materials in other engineering applications. For this reason the note investigates the application of the flow stress approach supported by API-579 standards (API, 2000) to ship hull ultimate strength assessment instead of the minimal upper yield stress in the iterative-incremental calculation as it is used by CSR. The aim of this comparison is to find out the potential reserve in ship hull girder ultimate strength with respect to the calculation procedure supported by the CSR, if any, as a consequence of various idealization models in the elastic-plastic approaches to ultimate strength assessment.

2. The application of ideal elastic-perfectly plastic material behavior model

Study of the plastic behavior of structural elements is concerned with the analysis of stresses and strains in the plastic as well as elastic ranges. It provides a more realistic estimate of the ultimate load carrying capacities of structures and gives a more complete understanding of the ultimate response of structural elements to external loads. The practical engineering uses several idealized models to study structural deformations due to material yielding in plastic region as it is summarized in the sequel.

a) The most frequently used approximation is the elastic-perfectly plastic model. The simple elastic-perfectly plastic model approximates the experimental engineering stress-strain curve, Fig.1 (a), commonly assuming that the plasticity occurs as the flow stress $f$ reaches the yield stress $y$, Fig. 1(b).

$$c_f = cy$$

(1)

This simple model neglects entirely the effect of work hardening being in this sense from the beginning conservative with respect to the ultimate strength assessment, Fig. 1. The results represent the minimal ultimate strength. Therefore the material flow stress $f$ is to be defined differently of the yield stress $y$ in order to compensate the effect of the work hardening till reaching the ultimate stress $u$.

b) Another elastic-perfectly plastic model assumes that plastic flow occurs at the stress level between the yield stress $y$ and ultimate stress $u$, Fig. 1 (c).

$$c_f = cy + c \cdot (c_u - cy)$$

(2)

The flow stress $f$ is the stress along one axis at a given value of strain that is required to produce plastic deformation.

The modification (2) enhances the simple elastic-perfectly plastic idealization by indirect approximation of the work hardening in the idealized model (1). The material work
hardening represents a potential reserve in ultimate strength assessment in (2) with respect to the simple elastic-perfectly plastic model (1) that will be investigated in the sequel.

There are some other idealized models using different approximation methods for the stress-strain relation.

c) The Elastic-Linear Work Hardening Model: In this model the elastic and plastic regions are approximated by two straight lines. The first line with $\varepsilon = \frac{\varepsilon_y}{E_p}$, a slope of $E_o$, which represents the elastic region and the second line $\varepsilon = \frac{\varepsilon_y}{E_p} (\varepsilon - \varepsilon_y)/E_p$, with slope $E_p$, which represents the plastic region, Fig. 1 (d).

d) The Elastic-Experimental Hardening model fits the experimentally obtained stress-strain curve more closely by modeling the work hardening region with an exponential curve $\varepsilon = k \varepsilon^n$.

The Ramberg-Osgood Model represents the stress-strain curve with a power function $\varepsilon = \frac{\sigma}{E} + a \left( \frac{\sigma}{b} \right)^n$, Fig. 1 (e).

3. The CSR recommendation for the ship hull girder ultimate strength check

In definition of functional requirements relevant to ship structure the ultimate strength calculations have to include ultimate girder capacity and ultimate strength of plates and stiffeners for ships equal or greater than 150 meters in length. The ultimate strength of the hull girder is to withstand the maximum vertical longitudinal bending moment obtained by multiplying the partial safety factor and the vertical longitudinal bending moment at $10^{-8}$ probability level (IACS, 2006) 1, 2.
3.1. Hull girder bending moment

The vertical hull bending moment $M$ in sagging and in hogging conditions, to be considered in the ultimate strength check of the hull girder, is to be obtained, in $kN\cdot m$, in intact, flooded and harbor conditions, from the following formula:

$$M = M_{SW} + \gamma_{W} M_{WV}$$

(3)

where:

$M_{SW}$, $M_{SW,F}$, $M_{SW,P}$ : Design still water bending moment, in $kN\cdot m$, in sagging and hogging conditions at the hull transverse section considered, to be calculated respectively in intact ($M_{SW}$), flooded ($M_{SW,F}$) and harbor ($M_{SW,P}$) conditions,

$M_{WV}$, $M_{WV,F}$, $M_{WV,P}$ : Vertical wave bending moment, in $kN\cdot m$, in sagging and hogging conditions at the hull transverse section considered, respectively in intact ($M_{WV}$), flooded ($M_{WV,F}$) and harbor ($M_{WV,P}$) conditions,

$\gamma_{W}$ : Safety factor on wave hull girder bending moments, taken equal to $\gamma_{W} = 1.20$.

The ultimate bending moment capacities of a hull girder transverse section, in hogging and sagging conditions, are defined as the maximum values of the curve of bending moment capacity $M$ versus the curvature $\chi$ of the transverse section considered. The curvature $\chi$ is positive for hogging condition and negative for sagging condition. The curve $M-\chi$ is to be obtained through an incremental-iterative procedure, according to the criteria specified by the CSR 1, 2.

The vertical hull bending moment $M$ is to be checked that the hull girder ultimate bending capacity at any hull transverse section is in compliance with the following formula:

$$M \leq \frac{M_{U}}{\gamma_{R}}$$

(4)

where:

$M_{U}$ : Ultimate bending moment capacity of the hull transverse section considered, calculated with net offered scantlings based on gross offered thickness reduction, in $kN\cdot m$:

$M_{U} = M_{UH}$ in hogging conditions,

$M_{U} = M_{US}$ in sagging conditions,

$M_{UH}$ : Ultimate bending moment capacity in hogging conditions, in $kN\cdot m$

$M_{US}$ : Ultimate bending moment capacity in sagging conditions, in $kN\cdot m$,

$M$ : Bending moment, in $kN\cdot m$, for the ship in intact, flooded and harbor conditions,

$\gamma_{R}$ : Safety factor taken equal to 1.10.

3.2. Elastic-plastic collapse of structural elements

The equation describing the load-end-shortening curve for the ideal elastic-plastic collapse of structural elements composing the hull girder transverse section is to be obtained from the following formula, valid both for positive (shortening) ad negative (lengthening) strains:

$$\sigma = \Phi \cdot R_{eh}$$

(5)

The edge function $\Phi$ in (5) is defined as:
\[ \Phi = -1 \quad \text{for} \quad \varepsilon < -1 \]
\[ \Phi = \varepsilon \quad \text{for} \quad -1 < \varepsilon < 1 \]
\[ \Phi = 1 \quad \text{for} \quad \varepsilon > 1 \]

The relative strain is defined as \( \varepsilon = \frac{E}{E_Y} \) and the strain at yield stress is equal to \( \varepsilon_Y = \frac{R_{eH}}{E} \).

\( R_{eH} \) in (5) is the minimum yield stress \( (\gamma) \), in N/mm\(^2\), of the material and \( E \) is the Young’s modulus, in N/mm\(^2\), to be taken equal to \( E = 2.06 \times 10^5 \) N/mm\(^2\), for steels in general.

3.3. Beam column buckling, torsional buckling, web local buckling of ordinary stiffeners made of flat bars, plate buckling

The ultimate bucking strength checking does not account for material ultimate stress and employs only the minimal yield stress in two ways:
- to determine the criterion (Johnson-Ostenfeld’s parabola) for elastic and plastic buckling behavior
- to determine the effective width of attached shell plating

Note how the flow stress (5) does not affect directly the ultimate bucking strength checking only indirectly whether the buckling occurs in elastic or plastic region of material behavior.

4. API-579 "Engineering Failure Analysis" standards

The flow stress can be thought of as the effective yield strength of a work hardened material. The use of a flow stress concept permits the real material to be treated as if it were an elastic-plastic material which can be characterized by a single strength parameter. The flow stress can be used, for example, as the stress level in the material that controls the resistance of structure to failure by plastic collapse.

Several relationships for estimating the flow stress have been proposed by API-579 (API, 2000) 3:

1. The average of the yield and tensile strength (recommended for most assessments).
   \[ \sigma_f = \sigma_y + \frac{\sigma_u - \sigma_y}{2} \quad (6) \]
   Where in (6) \( \sigma_y \) is the yield stress and \( \sigma_u \) is the tensile stress.

2. The yield strength plus 69 MPa (normally the statistical mean value of the yield strength):
   \[ \sigma_f = \sigma_y + 69 \text{ MPa} \quad (7) \]

3. If Ramberg-Osgood parameters are available, the flow stress can be computed using the following equation.
\[
\sigma_f = \frac{\sigma_{ys}}{2} \left[ 1 + \left( \frac{1}{0.002n} \right)^{1/n} \right] \exp \left( \frac{1}{n} \right)
\]  

(8)

In the absence of a material test report for plate and pipe, and for weld metal, the specified minimum yield strength and the specified minimum tensile strength for the material can be used to calculate the flow stress. The mechanical properties of shipbuilding steels are summarized in Table 1.

5. The calculation procedure

The study investigates the current CSR 1, 2 iterative-incremental computational procedure with respect to various idealizations in elastic-plastic collapse assessments of structural elements.

The underlying idea of the study is to replace the minimal yield stress \( R_{eH} \) in the load-end-shortening formula (5) given in CSR with the flow stress \( f \) recommended by API-579 standards (6) as follows:

\[
\sigma = \Phi \cdot \sigma_f
\]

(9)

The hull girder ultimate strength assessment is then to be repeatedly performed with newly introduced values for flow stress \( f \) and compared to the results from the formerly executed CSR procedure 1, 2 with minimal yield stress values \( R_{eH} \).

6. Material mechanical properties

The mechanical properties of commonly used MS and HT shipbuilding steels are summarized next, Table 1.

| Steel | Yield stress | Ultimate stress | Mean(\(\sigma_u\)) | Flow stress 
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<tr>
<td></td>
<td>ReH min ((\sigma_y))</td>
<td>Rm ((\sigma_u))</td>
<td>+69MPa</td>
<td>(\sigma_f) = (\sigma_y + (\sigma_u - \sigma_y)/2)</td>
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<td>MS</td>
<td>235</td>
<td>400-520</td>
<td>304</td>
<td>317-377</td>
</tr>
<tr>
<td>HT32</td>
<td>315</td>
<td>440-570</td>
<td>384</td>
<td>377-442</td>
</tr>
<tr>
<td>HT36</td>
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<td>390</td>
<td>510-660</td>
<td>459</td>
<td>450-525</td>
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7. Examples

The following example reconsiders the ultimate strength for five tankers and three bulk carriers formerly assessed by using CSR 1, 2 recommended values of minimal shipbuilding steel yield stresses (IMAM, 2009) 5, Table 1. The results for elastic, fully plastic and ultimate bending moments for hogging and sagging conditions are obtained by using the computer program MARS (BV, 2003) 4 based on incremental-iterative algorithm, Table 2.

The ultimate strength assessment is repeatedly performed by using the API-579 3 recommended values for the flow stresses $f(6)$ instead of the minimal yield stress $ReH(5)$, using the minimal ultimate (or tensile) stress according to classification rules (value to the left in the third column), Table 1. The study focused on the ultimate bending strength reserve with respect to the CSR required safety factor $R$ equal to 1.10 relative to the design vertical bending moment $M$ of the ship hull(4), Table 3.

8. Conclusion

The study investigated how various methods of idealization of true material yielding behavior affect the ship hull girder ultimate strength assessments. The most frequently used elastic-perfectly plastic model that is also adopted by the CSR for tankers 2 and bulk carriers 1 approximates the experimental engineering stress-strain curve commonly assuming that plasticity occurs when the flow stress reaches the yield stress. Moreover, the CSR accept the minimal declared that is the nominal yield stress giving the minimal ship hull ultimate strength, which itself already introduces reserves with respect to the actual structural strength. The simple idealization model neglects entirely the effect of work hardening being in this sense conservative at the beginning. The API standards takes the material flow stress above the yield stress in order to compensate the effect of the work hardening till reaching the ultimate stress. The study therefore calculated the potential reserve in ship hull girder ultimate strength with respect to the simple elastic-perfectly plastic model used by CSR but now additionally accounting for material work hardening as it is adopted by application of modified flow stress recommended API standards. The calculation procedure in the study applied both for the CSR and for API approaches the iterative-incremental procedure as it is recommended by CSR. The calculation is performed using the computer program MARS 4, Table 2. Former investigations on recently built tankers and bulk carriers (IMAM, 2009) 5 indicated how the ultimate strength of ship hull girder abundantly satisfies the rule requirement of partial safety factor $R$ at least amounting to 1.1 with respect to the design bending moment, Table 3. The study presented herein shows that the ultimate strength could be considered even higher if the potential reserve due to shipbuilding steel work hardening is taken into account by modified elastic-perfectly-plastic model with flow stress instead of the minimal yield stress.
Table 2. Longitudinal strength data of considered tankers (T) and bulk carriers (B)

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<th>Items</th>
<th>T1</th>
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<th>T4</th>
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<td>Lpp</td>
<td>m</td>
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<td>258</td>
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<td>17.1</td>
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<td>126653</td>
<td>107160</td>
<td>65200</td>
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<td>6.41</td>
<td>9.75</td>
<td>8.85</td>
<td>6.89</td>
<td>3.64</td>
<td>4.52</td>
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<td>M_S</td>
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<td>0.65</td>
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**API-579**

\[ \sigma_f = \sigma_y + \left( \frac{\sigma_u - \sigma_y}{2} \right) \] N/mm^2

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<tr>
<th>Items</th>
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<td>0.67</td>
<td>0.62</td>
<td>0.76</td>
<td>0.72</td>
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The potential reserve of the ultimate strength measured with respect to the safety factor \( \gamma \) obtained according to CSR idealized model with minimal yield stress, Table 2, could increase in amount up to 17-20% for hogging and 13-17% for sagging for API based flow stress assumption between the yield and ultimate stress, Table 3. The reserve could be even greater if instead of the minimal values of yield and ultimate stresses their mean or upper values would be taken into calculation.

The relation between the hogging and sagging ultimate bending moments \( M_u/M_{uS} > 1 \), Table 2, indicate that buckling failures have expectedly less influence in the hogging conditions than in the sagging condition. The relations between the hogging \( M_u/M_p \) or sagging \( M_{uS}/M_p \) ultimate bending moments and the fully plastic moment can be considered as the yielding index that show how in the hogging condition about 87-94% of the ultimate bending moment is due the regular yielding while the rest refers to buckling and early yielding, if any, Table 2. In the sagging condition that ratio is different, where about 62-79% of failures are due to regular yielding and the rest of the ultimate bending moment refers to buckling and early yielding, if any, Table 2.
Table 3. Ultimate strength checking for tankers and bulk carriers

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<td>1.20</td>
<td>1.20</td>
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<td>1.17</td>
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<td>$\gamma_{R,H}$</td>
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<td>1.18</td>
<td>1.18</td>
<td>1.17</td>
<td>1.19</td>
<td>1.20</td>
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<tr>
<td>$\gamma_{R,H}$</td>
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The increase of the material flow stress above the yield stress in order to compensate the effect of the work hardening till reaching the ultimate stress $\sigma_u$ leads to higher ultimate strength of the hull girder. However, this increase is slightly lower, 17-20% for hogging and 13-17% for sagging conditions, Table 3, compared to the increase of the material flow stress (20% in the example). That arises from the fact that higher material flow stress assured higher resistance against yielding while it has small effect on the buckling. The relation $\gamma_{R,H}/\gamma_{R,S}$ in Table 3, confirms once again that buckling jeopardize the ship hull more in sagging than in hogging condition.

REFERENCES