GPDs from DVCS at LO and beyond

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Diffractive and electromagnetic processes at the LHC
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Outline

Introduction
Proton structure (PDFs, form factors . . . )
From deeply virtual Compton scattering (DVCS)
to generalized parton distributions (GPDs)

DVCS at LO and beyond
Conformal moments, Mellin-Barnes representation and
higher-orders
Learning from LO

Numerical results
Size of Radiative Corrections
Fitting GPDs to Data
3D image of a proton

Outlook
DVCS (spacelike, timelike), meson electroproduction . . .

Summary
Parton distribution functions

- Deeply inelastic scattering, \(-q_1^2 \to \infty\), \(x_{BJ} \equiv -q_1^2/(2p \cdot q_1) \to \text{const}\)

\[
\sum_X \gamma^* \rightarrow \begin{array}{c}
\gamma^* \\
x_p \\
p
\end{array} = \begin{array}{c}
\gamma^* \\
x_p \\
p
\end{array}
\]

- no information on spatial distribution of partons

Electromagnetic form factors

- Dirac and Pauli form factors:

\[
q(b_\perp) \sim \int dq_1 e^{iq_1 \cdot b_\perp} F_{1,2}(t = q_1^2)
\]

- “skewless” GPD: \(H^q(x, 0, t = \Delta^2) = \int db_\perp e^{i\Delta \cdot b_\perp} q(x, b_\perp)\)
Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller ‘92, et al. ‘94]

\[ \begin{align*}
\gamma^* & \rightarrow P_1 P_2 \\
-q_1^2 = Q^2 & \quad q_2^2 = 0 \\
\end{align*} \]

\[ P = P_1 + P_2 \]
\[ q = (q_1 + q_2)/2 \]

- There is a background process but it can be used to our advantage:

\[ \sigma \propto |T_{\text{DVCS}}|^2 + |T_{\text{BH}}|^2 + T_{\text{DVCS}}^* T_{\text{BH}} + T_{\text{DVCS}} T_{\text{BH}}^* \]

- Using \( T_{\text{BH}} \) as a referent “source” enables measurement of the phase of \( T_{\text{DVCS}} \)

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Deeply virtual Compton scattering

- Measured in leptoproduction of a real photon:

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Factorization of DVCS $\rightarrow$ GPDs

\[ -q_1^2 = Q^2, \quad q_2^2 = 0 \]

- Compton form factor is a convolution:

\[ ^aH(\xi, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ H^a(x, \eta = \xi, t, \mu^2) \]

\( a = NS, S(\Sigma, G) \)

- \( H^a(x, \eta, t, \mu^2) \) — Generalized parton distribution (GPD)

Properties of GPDs

- Forward limit (\( \Delta \rightarrow 0 \)): \( \Rightarrow \) GPD \( \rightarrow \) PDF

\[ F^q(x, 0, 0) = H^q(x, 0, 0) = \theta(x)q(x) - \theta(-x)\bar{q}(-x) \]

Sum rules:

\[ \sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{c} H^q(x, \eta, \Delta^2) \\ E^q(x, \eta, \Delta^2) \end{array} \right\} = \left\{ \begin{array}{c} F_1(\Delta^2) \\ F_2(\Delta^2) \end{array} \right\} \]

- Possibility of solution of proton spin problem

\[ \frac{1}{2} \int_{-1}^{1} dx \ x \left[ H^q(x, \eta, \Delta^2) + E^q(x, \eta, \Delta^2) \right] = J^q(\Delta^2) \]  
  [Ji ’96]
DVCS at LO and beyond

based on:

- [DVCS using Mellin-Barnes representation, going to higher-orders and fitting GPDs]
  D. Müller,
  *Next-to-next-to leading order corrections to deeply virtual Compton scattering: The Non-singlet case*, [hep-ph/0510109]
  K. Kumerički, D. Müller, K. Passek-K., A. Schäfer,
  *Deeply virtual Compton scattering beyond next-to-leading order: the flavor singlet case*, [hep-ph/0605237]
  K. Kumerički, D. Müller, K. Passek-K.,
  *Towards a fitting procedure for deeply virtual Compton scattering at next-to-leading order and beyond*, [hep-ph/0703179]

- [Getting the right information from LO]
  K. Kumerički, D. Müller, K. Passek-K.,
  K. Kumerički, D. Müller,
  *Deeply virtual Compton scattering at small x(B) and the access to the GPD H*, [arXiv:0904.0458 [hep-ph]]

\[ \mathcal{H}(\xi, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) H^a(x, \eta = \xi, t, \mu^2) \]

\( a = \text{NS, S}(\Sigma, G) \)

- C^a (hard-scattering amplitude i.e. Wilson coefficient):
  - LO, NLO (1st order in \( \alpha_s \))
    - [Ji et al, Belitsky et al, Mankiewicz et al, ’97]
      \( \Rightarrow \) need **NNLO** to stabilize perturbation series and investigate convergence

- \( H^a \) (GPD):
  - Complete deconvolution is impossible, so to extract GPDs from the experiment we need to **model their functional dependence**.
  - Evolution known to NLO order and not trivial to implement.
• factorization formula for singlet DVCS CFFs:

\[ S^H(\xi, t, Q^2) = \int dx \ C(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ H(x, \xi, t, \mu^2) \]

• ... in terms of conformal moments

(analogous to Mellin moments in DIS: \( x^n \rightarrow c_n^{3/2}(x), c_n^{5/2}(x) \)):

\[
H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} dx \ \eta^{j-1} C^3/2(x/\eta) H^q(x, \eta, \ldots)
\]

\[
\eta^j \text{ even polynomials in } \eta \text{ with maximal power } \eta^{j+1}
\]

• series summed using Mellin-Barnes integral over complex \( j \):

\[
= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \left[ i + \tan \left( \frac{\pi j}{2} \right) \right] \xi^{-j-1} C_j(Q^2/\mu^2, \alpha_s(\mu)) \ H_j(\xi, t, \mu^2)
\]

Advantages of conformal moments and Mellin-Barnes representation

• **NNLO corrections** accessible by making use of conformal OPE and known NNLO DIS results
• enables simpler inclusion of **evolution** effects
• possible efficient and stable numerical treatment \( \Rightarrow \) stable and fast **computer code** for evolution and fitting
• powerful analytic methods of **complex j** plane are available (similar to complex angular momentum of Regge theory)
• opens the door for interesting **modelling of GPDs**
• moments are equal to matrix elements of local operators and are thus directly accessible on the **lattice**
Modelling conformal moments

- \( \eta \)-dependence inspired by SO(3) partial wave decomposition of \( \gamma^* \gamma \rightarrow pp \) scattering (similar to “dual” parametrization [Polyakov, Shuvaev '02])

\[
H_j(\eta, t) = \left( N^\Sigma_F(t) B(1 + j - \alpha(0), 8) \right) + \left( N^G_F(t) B(1 + j - \alpha(0), 6) \right)
\]

Leading partial wave

- Leading wave (we have used in [hep-ph/0703179]):
  - Regge-inspired ansatz
  - for \( t = 0 \) corresponds to x-space PDFs of the form
    \[
    \Sigma(x) = N^\Sigma_x x^{-\alpha(0)} (1 - x)^7; \quad G(x) = N^G_x x^{-\alpha(0)} (1 - x)^5
    \]

- at NLO data can be fitted with leading wave only
- but at LO we need \( \eta \)-dependence!
  - included in the new LO analysis

Sum rules and GPDs from LO

[Teryaev '05; Kumerički, Müller and Passek-K. '07, '08; Diehl and Ivanov '07;]

- LO perturbative prediction
  \[
  \mathcal{H}(\xi, t, Q^2) \overset{LO}{=} \int_{-1}^{1} dx \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mu^2 = Q^2)
  \]

\[
\frac{1}{\pi} \Im \mathcal{H}(\xi = x, t, Q^2) \overset{LO}{=} H(x, \xi = x, t, Q^2) - H(-x, \xi = x, t, Q^2)
\]

- dispersion relation

\( \Rightarrow \) various sum rules . . .

The goal is to reveal from DVCS data the GPDs at their cross-over trajectory \( \eta(=\xi) = x \) and to obtain a generic understanding of the skewness effect.

\( \rightarrow \) model dependent extraction of \( H \) at the cross-over trajectory also for large \( \xi \) (JLab data) [Kumerički and Müller '09]
NLO and NNLO corrections

for generic parameters

\[ \delta^P K = \left| \frac{H^{NP \text{LO}}}{H^{NP-1 \text{LO}}} \right| - 1, \quad \delta^P \varphi = \arg \left( \frac{H^{NP \text{LO}}}{H^{NP-1 \text{LO}}} \right) \]

Fast fitting routine (GeParD)

• \[ \int d\xi C_j(Q^2) \times \mathcal{E}(Q^2, Q^0) \times H_j(Q^2_0) \Rightarrow \text{Observable} \]

• Check by comparison to QCD-PEGASUS [Vogt '04] and evolution of Les Houches benchmark PDFs
Fits (GeParD output)

NNLO fit to HERA DVCS+DIS data
Three-dimensional image of a proton

Quarks:

Gluons:

Complementary processes

(General) DVCS
\[ \gamma^* p \rightarrow \gamma^* p \]
\[ (ep \rightarrow epl^+l^-) \]
[Belitsky, Müller '02,'03; Guidal, Vanderhaeghen '02]

Spacelike DVCS
\[ \gamma^* p \rightarrow \gamma p \]
\[ (ep \rightarrow ep\gamma) \]

Timelike DVCS
\[ \gamma p \rightarrow \gamma^* p \]
\[ (\gamma p \rightarrow pl^+l^-) \]
[Berger, Diehl, Pire '01; Pire et al '08, Afanasev et al '09]

deeply virtual electroproduction of mesons (DVEM)

more difficult, but access to flavours
\[ \gamma^* p \rightarrow Mp \]

NLO: [Belitsky and Müller '01, Ivanov et al '04]
Timelike DVCS
\[ \gamma(q_1)p(P_1) \rightarrow \gamma^*(q_2)p(P_2), \quad q^2 = q_2^2 = Q^2 \rightarrow \infty \]

- experimentaly accesible from exclusive photoproduction of lepton pairs \( \gamma p \rightarrow l^+l^- p \)
- Bethe-Heitler amplitude much more important \( \rightarrow \) always bigger then DVCS
- offers relatively simple access to the real part of the Compton amplitude via the angular distribution of the lepton pair
- preliminary analysis was performed at LO [Berger, Diehl, Pire '01]
- in timelike regime important contributions and effects possible at higher-orders
- To do:
  - modification of [Belitsky and Müller '03] formulas for \( pp \rightarrow ppl^+l^- \) process relevant for, say, ALICE
  -applying the formalism and higher-order expressions developed for spacelike DVCS to the timelike case
  -analysis of the experimental feasibilities (see, e.g., [Pire et al '08])

Summary

- Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVEM ... different processes offer different insight and should provide more complete picture.
- Spacelike DVCS analyzed up to NNLO:
  - Using conformal moments of GPDs has several advantages, including
    - elegant approach to NLO and NNLO corrections
    - providing convenient framework for GPD modelling
  - NLO corrections can be sizable; NNLO corrections are small, supporting perturbative framework of DVCS; scale dependence is not so conclusive: large NNLO effects for \( \xi \lesssim 10^{-3} \)
  - Fits to available DVCS and DIS data also work well and give access to transversal distribution of partons; in order to get good LO fits, one needs more sophisticated GPD modelling.
  - The analysis of DVEM and timelike DVCS along the same lines is under way.

The End
Definition of GPDs

- In QCD GPDs are defined as [Müller ’92, et al. ’94, Ji, Radyushkin ’96]

\[
F^q(x, \eta, t = \Delta^2) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2|q(-z)\gamma^+ q(z)|P_1\rangle \bigg|_{z^+ = 0, z_\perp = 0}
\]

\[
F^g(x, \eta, t = \Delta^2) = \frac{4}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2|G_+^{i\mu}(-z)G_{a\mu}^+(z)|P_1\rangle |_{\mu = 0}
\]

Properties of GPDs

- Decomposing into helicity conserving and non-conserving part:

\[
F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2)i\sigma^{\nu\mu} u(P_1)\Delta_\nu}{2M_P^+} E^a \quad a = q, g
\]

- Forward limit ($\Delta \to 0$): $\Rightarrow$ GPD $\rightarrow$ PDF

\[
F^q(x, 0, 0) = H^q(x, 0, 0) = \theta(x)q(x) - \theta(-x)\bar{q}(-x)
\]

Sum rules:

\[
\sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{c}
H^q(x, \eta, \Delta^2) \\
E^q(x, \eta, \Delta^2)
\end{array} \right\} = \begin{array}{c}
F_1(\Delta^2) \\
F_2(\Delta^2)
\end{array}
\]

- Possibility of solution of proton spin problem

\[
\frac{1}{2} \int_{-1}^{1} dx \left[ H^q(x, \eta, \Delta^2) + E^q(x, \eta, \Delta^2) \right] = J^q(\Delta^2) \quad [\text{Ji '96}]
\]
• DVCS belongs to a class of two-photon processes (DIS, DVCS, two-photon production of hadronic states . . . ) calculable by means of OPE

\[ T_{\mu\nu}(q, P, \Delta) = \frac{i}{e^2} \int d^4x \, e^{ix\cdot q} \langle P_2, S_2 | Tj_\mu(x/2)j_\nu(-x/2) | P_1, S_1 \rangle \]

\[ \rightarrow C_j O_j \]

\[ \downarrow \]

generalized Bjorken kinematics
conformal symmetry
\{ \} \rightarrow \text{unified description}

Conformal OPE (COPE)

• COPE prediction for general kinematics reads

\[ C_j(\eta/\xi, Q^2/\mu^2, \alpha_s^* = \text{fixed}) = c_j(\alpha_s^*) \begin{pmatrix} 2 + 2j + \gamma_j(\alpha_s^*) \\ 5 + 2j + \gamma_j(\alpha_s^*) \end{pmatrix}/2 \begin{pmatrix} \mu^2 \\ \left(\frac{Q^2}{\xi^2}\right) \end{pmatrix} \gamma_j(\alpha_s^*) \]

• \( \eta = 0 \): DIS

\[ \lim_{\eta \to 0} C_j(\eta/\xi, Q^2/\mu^2, \alpha_s^*) = c_j^{DIS}(\alpha_s^*)|_{\beta = 0} \left(\frac{\mu^2}{Q^2}\right) \gamma_j(\alpha_s^*)/2 \]

• \( \eta = \xi \): DVCS

• \( \eta = 1 \): photon-to-pion transition form factor
Breaking of conformal symmetry

- massless QCD is conformally symmetric at the tree level
- conformal symmetry broken at the loop level (renormalization introduces mass scale)
  - running of the coupling constant $\Rightarrow \beta \neq 0$
  - renormalization of the composite operators
    $\Rightarrow$ non-diagonal anomalous dimensions $\gamma_{jk} = \delta_{jk} \gamma_j + \gamma_{j k}^{\text{ND}}$

\[
\mu \frac{d}{d\mu} O_j(..., \mu^2) = - \sum_{k=0}^{j} \gamma_{jk}(\alpha_s(\mu)) \eta^{j-k} O_k(..., \mu^2),
\]

\[
\mu \frac{d}{d\mu} C_j(..., Q^2/\mu^2, \alpha_s(\mu))] = \sum_{k=j}^{\infty} C_k(..., Q^2/\mu^2, \alpha_s(\mu)) \gamma_{kj}(\alpha_s(\mu)) \left( \frac{\eta}{\xi} \right)^{k-j},
\]

Conformal scheme

- non-diagonal terms of anomalous dimensions ($\overline{\text{MS}}$ scheme) can be removed by finite renormalization, i.e. specific choice of factorization scheme $\rightarrow$ conformal subtraction ($\overline{\text{CS}}$) scheme:

\[
C^{\overline{\text{MS}}} O^{\overline{\text{MS}}} = C^{\overline{\text{MS}}} B B^{-1} O^{\overline{\text{MS}}} = C^{\overline{\text{CS}}} O^{\overline{\text{CS}}}
\]

\[
\gamma_{jk}^{\overline{\text{CS}}} = \delta_{jk} \gamma_j + \frac{\beta}{g} \Delta_{jk}
\]

- however, there is ambiguity in $\overline{\text{MS}} \rightarrow \text{CS}$ rotation matrix:

\[
B = B^{(\beta=0)} + \frac{\beta}{g} \delta B
\]

and by judicious choice of $\delta B$ one can “push” mixing to NNLO ($\overline{\text{CS}}$ scheme, [Melić et al. ’02] $\rightarrow \Delta_{jk}$ — unknown correction, starts at NNLO, and can be suppressed by choice of initial condition — neglected
NNLO DVCS

• Finally

\[ C^{CS, DVCS}_j(Q^2/\mu^2, \alpha_s(\mu)) = C_k(1, \alpha_s(Q)) \mathcal{P} \exp \left\{ \int_Q^\mu \frac{d\mu'}{\mu'} [\gamma_j(\alpha_s(\mu')) \delta_{kj}] \right\} \]

with

\[ C_j(1, \alpha_s(Q)) = \frac{2^{1+j+\gamma_j(\alpha_s)/2} \Gamma(\frac{5}{2} + j + \gamma_j(\alpha_s)/2)}{\Gamma(3/2)\Gamma(3+j+\gamma_j(\alpha_s)/2)} c_{j, MS, DIS}^{\alpha_s} \]

• we take

\[ c_{j, MS, DIS}^{\alpha_s}(\alpha_s) \text{ from [Zijlstra, v. Neerven ‘92, ‘94, v. Neerven and Vogt ‘00] } \]

\[ \gamma_j \text{ from [Vogt, Moch and Vermaseren ‘04] } \]

Conformal algebra

• Conformal group restricted to light-cone \( \sim O(2, 1) \)

\[ L_+ = -iP_+ \quad [L_0, L_\pm] = \mp L_\pm \quad \text{conf. spin } j: \]

\[ L_- = \frac{i}{2} K_- \quad [L_-, L_+] = -2L_0 \quad [L^2, \Omega_{n,n+k}] = \]

\[ L_0 = \frac{i}{2} (D + M_{+-}) \quad L^2 = L_0^2 - L_0 + L_-L_+ \]

\( D \quad - \text{ dilatations, } K_- \quad - \text{ special conformal transformation (SCT)} \)
Operator Product Expansion

\[ J_{\text{em}}(x)J_{\text{em}}(0) \longrightarrow \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \left( \frac{1}{x^2} \right)^2 x^{n+k+1} C_{n,k} O_{n,k} \]

\[ k = 0 : \quad O_{n,0} \equiv \bar{\psi} \gamma^+(i \leftrightarrow D_+)^n \psi \quad \text{i.e.} \quad i\partial_+ \stackrel{\text{M.E.}}{\rightarrow} -\Delta_+ \]

- \( C_{n,0} \) and \( \gamma_n \) of \( O_{n,0} \) are well known from DIS up to NNLO.
- But \( C_{n,k} \) and \( \gamma_{n,k} \) are not so well known.
- \( \gamma_{n,k} \neq 0 \Rightarrow \) operators \( O_{n,k} \) mix under evolution.
- Choosing operator basis in which \( \gamma_{n,k} \) is diagonal would help. But how to diagonalize unknown matrix?!
- (At least) to LO answer is: use conformal operators.

Conformal operators

\[ \mathcal{O}_{n,n+k} = (i\partial^+)^{n+k} \bar{\psi} \gamma^+ C_n^{3/2} \left( \frac{\bar{D}_+}{\partial^+} \right) \psi \]

- they have well-defined conformal spin \( j = n + 2 \)
- massless QCD is conformally symmetric at the tree level \( \Rightarrow \) conformal spin is conserved
- mixing of operators with different \( n \) is forbidden by conformal symmetry, while mixing of those with different \( n + k \) is forbidden by Lorentz symmetry \( \Rightarrow \mathcal{O}_{n,n+k} \) don’t mix at LO
- conformal symmetry broken at the loop level (renormalization introduces mass scale, dimensional transmutation) \( \Rightarrow \quad \mathcal{O}_{n,n+k} \) start to mix at NLO
• Diagonalize in artifical $\beta = 0$ theory by changing scheme

$$\mathbf{\Omega}^{\text{CS}} = B^{-1} \mathbf{\Omega}^{\overline{\text{MS}}}$$

so that

$$\gamma_{jk}^{\text{CS}} = \delta_{jk} \gamma_k$$

• $C_{n,k} = (-1)^k \frac{(n+2)_k}{k!(2n+4)_k} C_{n,0} \Rightarrow$ summing complete tower

$\beta \neq 0$ (I)

• In full QCD $\beta \neq 0$ and NLO diagonalization is spoiled:

$$\gamma_{jk}^{\text{CS}} = \delta_{jk} \gamma_k + \frac{\beta}{g} \Delta_{jk}$$

• However, there is also ambiguity in $\overline{\text{MS}} \rightarrow \text{CS}$ rotation matrix:

$$B = B^{(\beta=0)} + \frac{\beta}{g} \delta B$$

• By judicious choice of $\delta B$ one can “push” mixing to NNLO ($\overline{\text{CS}}$ scheme, [Melić, Müller and Passek ’02]).

• But how to calculate rotation matrix $B$? This is problem equivalent to calculation of $\gamma_{j,k}$. 

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\[ \beta \neq 0 \ (\text{II}) \]

- The \( B(\beta=0) \) is constrained by conformal Ward identities ...

\[
B_{jk}^{(\beta=0)\text{NLO}} = \delta_{jk} - \frac{\alpha_s}{2\pi} \theta(j > k) \frac{\gamma_{jk}^{\text{SCT, LO}}}{a_{jk}}
\]

\( \gamma_{jk} \) — known matrix

\[ [\text{Müller '93}] \]

\[ \text{SCT} \equiv \text{special conformal transformation} \]

- ... and, as a consequence

\[
\bar{M}^{\text{ND}, (1)} = \frac{\gamma_{\text{SCT}, (0)}^{\text{SCT, LO}} - \beta_0 b_g, \gamma^{(0)}}{a_{jk}}
\]

- Final result:

\[
\text{n-loop DIS (diagonal) result} + (n-1)\text{-loop SCT anomaly} = \text{n-loop non-diagonal prediction}
\]

Mellin-Barnes representation of CFFs

\[
S_{\mathcal{H}}(\xi, t, Q^2) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \left[ i + \tan \left( \frac{\pi j}{2} \right) \right] \xi^{1-j} \mathcal{C}_j(Q^2/\mu^2, \alpha_s(\mu)) H_j(\xi, t, \mu^2)
\]
• We have used previously developed formalism to
  1. investigate size of NNLO corrections to non-singlet
     and singlet Compton form factors in $\overline{\text{CS}}$ scheme
  2. compare the $\overline{\text{CS}}$ NLO predictions to complete $\overline{\text{MS}}$ NLO
     predictions (non-diagonal evolution included) and analyze the
     hep-ph/0703179]
  3. perform fits (in both schemes) to DVCS (and DIS) data and
     extract information about GPDs [K. Kumerički, D. Müller, K. P-K.,
  4. analyze the new HERA data to LO (investigating $\eta$-dependent

Check

• Check by comparison to QCD-Pegasus [Vogt ’04]
• evolution of Les Houches benchmark PDFs:
**Parton probability density**

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on $x$ and transversal distance $b$
  
  \[ H(x, \vec{b}) = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} e^{-i\vec{b} \cdot \vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2) , \]

- Average transversal distance:
  
  \[ \langle \vec{b}^2 \rangle(x, Q^2) = \frac{\int d\vec{b} \vec{b}^2 H(x, \vec{b}, Q^2)}{\int d\vec{b} H(x, \vec{b}, Q^2)} = 4B(x, Q^2), \]

  (at $Q^2 = 4 \text{ GeV}^2$)

  \[ \langle \vec{b}^2 \rangle_{\text{gluon}}(\xi = 10^{-3}) = 0.30^{+0.07}_{-0.04} \text{ fm}^2 \]
Beam charge asymmetry

\[ BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} = \frac{A_{\text{Interference}}}{|A_{\text{DVCS}}|^2 + |A_{\text{BH}}|^2} \propto \frac{F_1 \mathcal{H} + |t|}{4M^2} F_2 \mathcal{E} \]

- Model \( E_{\text{sea}} \) as \( \kappa_{\text{sea}} H_{\text{sea}} \) and take \( B_{\text{sea}} \equiv \int dx xE_{\text{sea}} \) as parameter

- H1 data enable exclusion only of very negative \( B_{\text{sea}} \)

Modelling conformal moments of GPDs

- How to model \( \eta \)-dependence of GPD’s \( H_j(\eta, t) \)?
- Idea: consider crossed \( t \)-channel process \( \gamma^* \gamma \to pp \)

When crossing back to DVCS channel we have:

\[ \cos \theta_{\text{cm}} \rightarrow -\frac{1}{\eta} \]

- \( d_{0,\nu}^J \) — Wigner SO(3) functions (Legendre, Gegenbauer, \ldots)
- Similar to “dual” parametrization [Polyakov, Shuvaev '02]
Modelling conformal moments of GPDs (II)

- OPE expansion of both $\mathcal{H}$ and $\mathcal{H}^{(t)}$, as well as trivial crossing properties of Wilson coefficients $C_j$, leads to

$$H_j(\eta, t) = \eta^{j+1} H_j^{(t)}(\cos \theta = -\frac{1}{\eta}, s(t) = t)$$

- and $t$-channel partial waves are modelled as:

$$H_j(\eta, t) = \sum_{j} h_{j,j} \left( \frac{1}{m(J) - t} - \frac{1}{J - \alpha(t)} \right) \eta^{j+1}$$

- Similar to “dual” parametrization [Polyakov, Shuvaev '02]

Skewness effect (I) — $R$

$$R \equiv \frac{\text{Im} A_{\text{DVCS}}}{\text{Im} A_{\text{DIS}}} \bigg|_{t=0} \propto \frac{H(\xi, \xi)}{H(2\xi, 0)} \quad \text{[Shuvaev et al. '99]}$$

- Significant skewness effect?
Skewness effect (II) — $r$

- Skewness effect is naturally defined by ratio of GPDs $H(x, \eta)$ at two physically relevant trajectories: $\eta = x$ and $\eta = 0$

$$r = \frac{H(x, x)}{H(x, 0)} \overset{LO}{\approx} \frac{1}{2^\alpha} R \quad \text{for} \quad q(x \to 0) \sim x^{-\alpha} \quad \alpha \approx 1$$

Skewness effect (III)

- To get the correct normalization and $t$-dependence, one has to compensate “natural” DVCS-to-DIS enhancement factor
  
  \[ \frac{2^{j+2} \Gamma(j + 5/2)}{\sqrt{\pi} \Gamma(j + 3)} \bigg|_{j = \alpha - 1 \approx 0.2} \approx 1.5 \]

- at NLO radiative corrections take care of that
- at LO resummed subleading partial waves have to give negative contribution:

\[ H_j(\eta, t) = \left( \frac{N_f}{N_G} F_\Sigma(t) B(1 + j - \alpha_\Sigma(0), 8) \right) + \left( \frac{s_{\Sigma}}{s_G} \right) \left( \begin{array}{c} \text{subleading partial waves, } \eta^- \text{-dependence!} \\ \text{negative “intrinsic skewness”} \end{array} \right) \]
GPDs and sum rules

[Teryaev ’05; Diehl and Ivanov ’07]

• LO perturbative prediction

\[ \mathcal{H}(\xi, t, Q^2) \overset{\text{LO}}{=} \int_{-1}^{1} dx \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mu^2 = Q^2) \]

\[ \frac{1}{\pi} \Im \mathcal{H}(\xi = x, t, Q^2) \overset{\text{LO}}{=} H(x, \xi = x, t, Q^2) - H(-x, \xi = x, t, Q^2) \]

\[ \Re H(x, \xi, t, Q^2) \overset{\text{PV}}{=} \int_{0}^{1} dx \left( \frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x} \right) H^{-}(x, \xi, t, \mu^2 = Q^2) \]

⇒ various sum rules
One strategy for extracting GPD information to LO accuracy:
- model the $t$-dependency of the GPD at $\eta = 0$
- parameterize the skewness function $S$ defined in

$$H^-(x, x, t, Q^2) = \left[1 + S(x, t, Q^2 | H^-)\right] H^-(x, \eta = 0, t, Q^2).$$

- fit parameters to measured observables (CFFs)

The goal of this LO analysis and of this concept is to reveal GPDs at their cross-over trajectory $\eta = x$ from DVCS data and to obtain a generic understanding of the skewness effect.

More work in this direction needed...

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**Relevance of GPDs for collider physics**

- heavy particle production $\Rightarrow$ collision is more central $\Rightarrow$ larger probability for multiple parton collisions

  - [Frankfurt, Strikman and Weiss ’04]

- No influence on total inclusive cross sections, but **event structure** is sensitive to transversal parton distributions.

- Relevant for LHC?