

HEATING SYSTEM IN THE ANCIENT WORLD: THE EXAMPLE OF THE SOUTHWESTERN *BALNEUM* IN DIOCLETIAN'S PALACE IN SPLIT

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ABSTRACT

Searching for historical backgrounds of today's technological achievements, the authors examined a case of heat transfer in the Roman period. This paper reports on an interdisciplinary investigation of the properties of the heating system of the private baths in the imperial palace of the roman emperor Diocletian (end of 3rd - beginning of the 4th C. AD) in Split (Croatia). The analysis of temperature distribution in those baths helped archaeologists to determine the functions of each of the rooms in the premises. On the other hand, thermodynamic calculation was conducted in order to get an insight in heating efficiency and temperature distribution in the most preserved part of the *thermae*.

INTRODUCTION

Everyday life and topics related to microhistory have become practically the most important issue of any research in the history, art history and archaeology for the last few decades. That growing interest on the subject includes the technologies of remote times. The social component, the technical one as well, are slowly becoming priorities in investigation. Unfortunately, archaeologists are usually not so familiar with the technological aspects and functioning of their discoveries, so they often remain only partially explained. In this matter interdisciplinary research is a necessity, and is becoming more and more a standard procedure.

Let us put forward just an example considering the heat transfer. Archaeologists are perfectly aware of the heating systems in the Roman world. There have been attempts, in cooperation with engineers, to understand the heating systems in Roman houses, there have even been attempts of measurement of temperatures in those houses, based on archaeological finds, written sources from the Roman period and experiments. But when we consider a specific topic, the

roman baths (*thermae*), calculations of that sort have seldom been tried, mostly due to a lack of interdisciplinary cooperation, but on the other hand also because of the lack of definitive data on the considered antique monuments. Therefore, in any calculation of the sort there are a number of hypotheses to take into consideration, and bear in mind that any interpretation might have to be corrected by further archaeological evidence. Nevertheless, this is an attempt, aside pure theoretical, already conducted, that tries to deal with a real case of heat transfer in a roman *thermae*, the one in Diocletian's palace in Split (Croatia).

HEATING SYSTEMS IN ROMAN BATHS

In past two decades knowledge about the Roman private and public baths has been significantly improved. Attention was turned from the renowned large scale baths in the city of Rome to the more modest scale baths which were part of an almost universal culture of the Roman world. Various issues related to this culture and the accompanying architecture have been debated from the end of the 19th century, starting from the role of the baths in everyday life in the Roman world to the reconstruction of the proper sequences of usage of the baths' apartments [1-8]. Among these, the one related to the heating system attracted the utmost attention since the distribution of heat in Roman *thermae* and *balnea* was often the only way to determine functions of individual rooms which were attested by scant archaeological remains. The so-called "hypocaust system", an original Roman invention, allowed for the distribution of hot air through a system situated under the elevated floor, suspended by *pilae* (small pillars made of bricks or stone) over the chambers positioned underneath of the baths' apartments. In majority of cases, depending on the position of the combustion chamber (*praefurnium*), position of the *caldarium* (the room with the hot bath) could be ascertained,

considering that these two, almost without an exception, were directly connected to provide the influx of the hottest air beneath the *caldarium* and a pool in which the highest temperature was desired. Typological studies have shown that the close connection of the *praefurnium* with the *caldarium* could be considered as a defining element for the interpretation of the overall layouts of many Roman baths. Of course, there are several other indicators that are equally instructive, such as the sizes of apertures between the various rooms allowing for the influx of regulated quantity of hot air into the *tepidarium* (tepid room), *sudatorium* (room serving as a dry sauna), or any other room, as well as the thickness of the floor.

Introduction of the hypocaust system to Roman baths and homes, urban and rural, undoubtedly represented one of the great inventions of the Roman civilization which transformed the standard of living on the Apennine peninsula, but even more in the northern provinces which were exposed to significantly colder weather throughout the year. However, although we are nowadays relatively well acquainted with the typological differences between various *balnea*, functions of different rooms in *balnea*, and with other architectural aspects of these structures, we know little about the actual ambient properties of the *caldarium*, *tepidarium* or *sudatorium*. There were many speculations and several experiments with hypocaust heating system in the past hundred years [9, 10]. Experiments were mostly limited to small sections of private bathing infrastructures and were thus inconclusive and inadequately instructive [9]. Equally confusing are the speculations about the temperature in rooms and pools of the Roman *caldaria* often cited in the case of the famous examples such as the hot room and *sudatorium* of the “imperial villa” near Piazza Armerina on Sicily.

DIOCLETIAN’S THERMAE – ARCHITECTURAL ANALYSIS

Precisely for this reason, we have selected an “imperial” example as the subject of an analysis in order to determine the values and properties of the heating system in one of the most representative, but still private Late Antique monuments. The chosen example is the western *balneum* of Diocletian’s palace in Split (Croatia), explored by archaeologists on several occasions, but never properly interpreted. Located in the southwestern part of the emperor’s retiring palace in his native province of Dalmatia, they were one of the two bathing complexes in the Palace built at the end of the 3rd and the beginning of the 4th century (Figure 1). Both of these thermal complexes, the eastern one whose layout is only partially reconstructed and the western one whose outlines are better explored, must have been a part of the original design of the imperial palace, although it was argued by some that their somewhat awkward position between the imperial apartments and the imperial mausoleum, in the case of the eastern bath, or *temenos* with the temples, in the case of western ones, could indicate that they were later interpolations [11, 12]. Authors who argued that the baths were built after the building of the Palace based their argumentation primarily on impressions of the position of baths in the overall plan of the imperial Palace. Main objection against an early date was the

assumption that the two *balnea* must have blocked the servant’s entrances to the imperial apartments situated in the southernmost part of the Palace [11]. However, there are no arguments for the assumption that the servant’s routes lead this way nor is there any reason for such an assumption. Contrary to these impressions, the layouts of the two *balnea* show that they were skillfully integrated into the only available space situated close to the imperial apartments. Z or L shaped plan was actually quite an imaginative solution to the problem of limited space. Forms and shapes of the rooms, especially ones of the western *balneum*, as will be explained, are probably the strongest argument against such assumptions. It was also suggested that the *balnea* in Diocletian’s palace were rather small and that their modest proportions certainly disqualified them from being “imperial” since the authors compared them, quite wrongly, with the huge public *thermae* built by emperors in Rome. Quite the opposite, these *balnea* were among the largest and most elaborated in the category of private baths intended for only a very limited number of bathers. There are several indicators that both of them were intended for only four bathers using them at the same time, so it would not be presumptuous to claim that their size and splendor by far exceeded their capacity. However, in both complexes remains of the luxurious interior decoration, such as glass *tesserae* sheeted with gold and expensive imported marbles, were found, indicating that both structures were built as an integral part of the imperial palace [13, 14]. What is even more important, it was also pointed out that the floor levels of both complexes follow the logic consistently implemented in all of the surrounding structures that were undoubtedly built as a part of the original concept [15].

It should be noted that the two *balnea* were adjacent to the parts of the palace which were recognized as the living quarters intended for the Emperor and Empress. Thus, it could be assumed that one of them was intended for the Emperor and his male entourage while the other was built to be at the disposition of the Empress and her suite. Such an arrangement certainly reflected the Roman tradition of separation of sexes in baths, attested in a number of Roman towns. This custom of mixing sexes in the baths was forbidden by the emperor Hadrian and by Marcus Aurelius. Alexander Severus prohibited any baths common to both sexes (*balnea mixta*) from being opened in Rome.

The western *balneum* of Diocletian’s palace (Figure 2) was explored in several archaeological campaigns [15]. The results of the last one, whose aim was to revise the situation documented in a campaign conducted at the beginning of 1970^s, have shown that the complex had a more elaborated layout than it was imagined after the previous explorations [12]. In fact, it was discovered that the complex consisted of at least eight rooms which covered the area of at least 180 m². Approximate sizes of rooms are: room 1 = 23.28 m² (with the apse), room 2 = 20.4 m², room 3 = 25 m² (without the apse), room 4 = 19.625 m² (without the 4 niches), room 5 = 31 m², room 6 = 56.56 m². It is hard to make a definite conclusion about the precise size of the rooms 7 and 8 since the accurate plans have not been published yet. Room 7 was not smaller than 18 m² and the room 8 was not smaller than 9 m². The

rooms were arranged in a Z-shaped plan which was obviously conditioned by the position of the *balneum* between the *temenos* of the temple of Jupiter and the back wall of the imperial apartments. According to I. Nielsen, the plan could be sorted under the “row type” of *balnea* with a double rotation of the main axis under 90 degrees [1].

Explorations of the complex in 1972 made it clear that the hypocaust extended beneath the majority of rooms. It was found under rooms 1, 3, 4, 5 and 6, while the existence of the furnace adjoining the room no. 5 was assumed with a high degree of certainty based on the size of the aperture found beneath the level of the floor in the apse of the same room. This arched opening, 1.5 meter high and 1.05 meter wide, found underneath the wall of the apse in the room no. 5, was labeled with a Greek letter α and was one of the two biggest apertures

connecting the hypocaust chambers under the floor (Figure 3). It is interesting that the arched opening designated as ϵ , connecting the rooms 3 and 7, had the same dimensions as α . On the other hand, two smaller openings were also found in the room no. 5. The one designated as β connected the chambers beneath the floors of the rooms 5 and 6. It was 1.19 meter high and 107 centimeters wide. Third opening in the room no. 5, situated beneath the wall dividing the rooms 5 and 4 (γ), was lower than the previous two (62.5 centimeters), as well as narrower (80 centimeters). Aperture connecting the chambers under the rooms 4 and 3 (δ) was of similar dimensions as γ . However, the aperture connecting the chambers underneath the rooms 3 and 1 was quite smaller, being only 50 cm high and 30 wide.

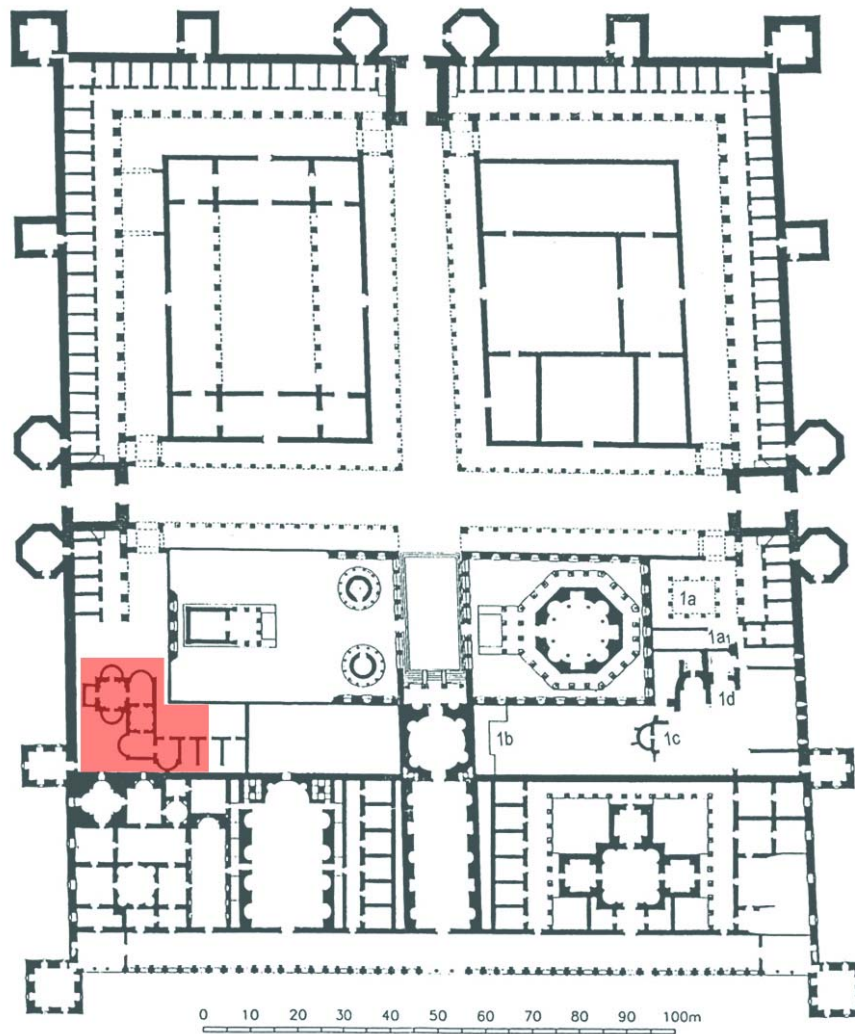


Figure 1 Plan of Diocletian's palace in Split and the positioning of the balneum

This arrangement of apertures beneath the floors, when compared with the standard solutions used throughout the Empire, clearly points to the fact that the furnace must have been situated on the other side of the wall of the apse in the

room no. 5. After all, this aperture was complemented by the two 75 cm high stone slabs positioned in a way to direct the hot air into the chamber beneath the room no. 5. This should be a clear indicator of the position of the furnace. But, it would be

strange that the *balneum* of this kind have had only one furnace and not several ones as it would be expected in the case of a luxurious private baths.

Thus, the sequence of rooms with different functions and properties becomes apparent through the analysis of the heating system situated below. The hottest room was undoubtedly the room no. 5 which was directly connected to the *praefurnium*. The hottest segment of this room was the apse situated exactly over the main, and possibly the only channel connecting the baths with the furnace. Thus, the room no. 5 should be identified as the *caldarium* with a hot bath tub or basin (*alveus*, *labrum*, *calda lavatio*, *balineum*, *calda piscina*), probably situated in the apse and made of some of the precious material.

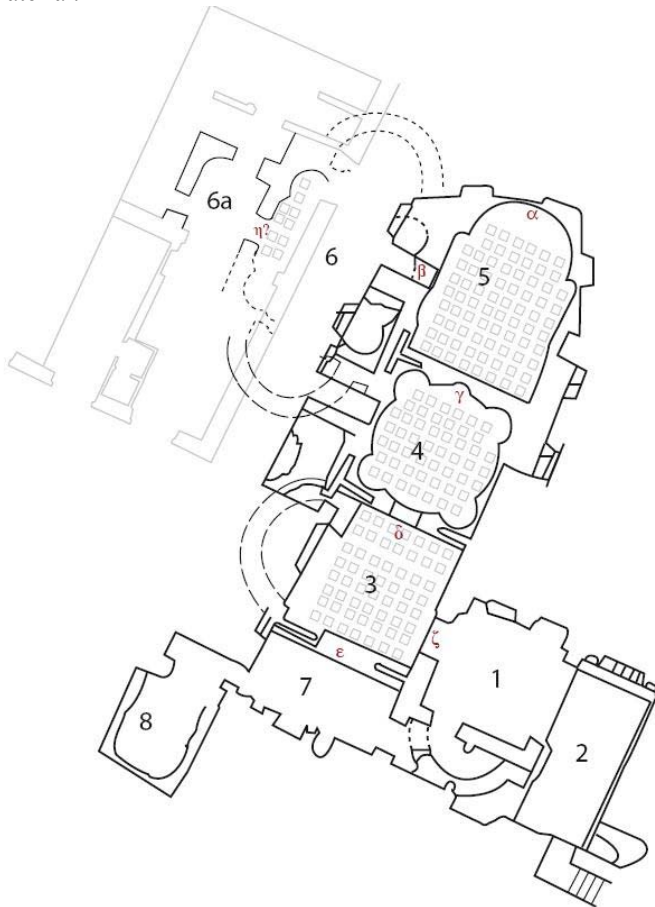


Figure 2 Plan of the *balneum*

However, the room no. 6 presents itself as the most interesting case for the reconstruction of the hypocaust system and for the interpretation of the architectural properties of the building as a whole. This room of an intriguing layout was subjected to the severest devastations during the 20th century when modern day cement walls and installations destroyed its middle part, protruding through its southern apse and obscuring almost half of its layout. Fortunately, enough remains were preserved from which the plan can be reconstructed. It was the largest and most elaborate room of the complex. Its core plan

was an octagon with four niches, each being wide 1.5 meter, of which three were attested by archaeological explorations. On the southern side it had an apse attached with a diameter of some 3.3 meter and it was correctly assumed by the researchers that there must have existed an identical one on the opposite side of the room (Figure 2). Altogether, the room covered 56.56 m², if we count in the potential northern apse. Next to the fact that it was situated beside the *caldarium*, a hypocaust chamber was found in this room thus also pointing to the fact that the room was heated. Moreover, the hypocaust chamber underneath the room was connected to the one under the room no. 5 by a quite sizable arched opening. So, it seems that this room must have been a hot room with two hot water basins.

Still, there are several other questions that can be raised regarding the room no. 6 which certainly reflect themselves on the reconstruction of the ambiental properties of the room and the entire complex. First of all, its layout suggests that the design of the room was adjusted to two functions. Central plan with niches, most certainly topped with a dome, suggest that the room was intended to be used as a *sudatorium*, or a sweating room. Niches were undoubtedly intended for sitting during the prolonged stay in the room. The same arrangement is attested in several other baths. However, two apses would be meaningless if they were not used as places for the basins which, judging by the existence of the hypocaust system beneath, must have been filled with hot water. Thus, the room must have been a combination between *caldarium* and *sudatorium* which were joined into one, either for the pleasure of the owner or because of the limited space between the imperial apartments, western wall of the whole Palace and the *temenos* of the temple

Secondly, there is a question of how was this hot room actually heated. It could be argued that the most of the heat entered from the room no. 5 through the sizable aperture connecting the two hypocaust chambers. But, it is equally probable that the room 6a, attached to the western wall of the room 6, served as a second *praefurnium* which complemented the work of the furnace behind the apse of the room no. 5 or, depending on the atmospheric conditions, could have been alternated with it. Room 6a was most probably 8.75 m² large and did not have a hypocaust (Figure 2). Its floor was 3.84 meter above the level of the sea, or, in other words, on more or less the same level as were the floors of the hypocaust chambers in adjacent rooms. Apparently there was no hypocaust in room no. 6a, but it was connected with the room no. 6 [16]. Thus, it seems plausible that the room served as a *praefurnium* whose position should have been below the level of the floor of the rooms elevated by hypocaust chambers. Thus, it is highly unlikely that it served as the unheated basin, which would be more than 2 meters deep, in a hot room. It is much more probable that the room 6a was indeed a second *praefurnium* which was delivering the hot air to the room no. 6 through an aperture whose existence is documented on the plans made in 1972 [16]. Furthermore, if the room 6a was indeed a *praefurnium*, it was situated in the perfect position, just next to the western wall of the Palace where it was not in a way to anybody and was not immediately noticeable (Figure 1).

If we are correct in the assumption about the double function of the room no. 6, than it could be further assumed that the room was built more or less according to Vitruvius' recommendations. The core of the room with its conspicuous octagonal shape with 4 niches certainly makes it likely candidate for the *laconicum*. The plan of this room has been reconstructed in several ways. In their publication summarizing the results of the explorations conducted in the late 1970', the authors have noted just one apse, the southern one. However, in his reconstruction of the overall plan of the palace, J. Marasovic has added another apse on the northern side, thus making the plan of the room symmetrical. Except of the small parts of an eastern niche, the room was not further explored recently since it was in most part ruined by the modern day cement walls and installation. But, the reconstruction offered by J. Marasovic appears to be altogether convincing and plausible. The room obviously had two apses symmetrically arranged, along with the four niches (Figure 2) [13, 16]. Vitruvius implied that the *laconicum*, or *sudatorium* as Seneca called it, should be a domed room with a central plan whose walls had a height from the floor to the bottom of the dome equal to the width of the room [16]. If we assume that the architects of Diocletian's palace followed Vitruvius' recommendations in a certain degree, in the case of the room no. 6, which indeed had an elaborate central plan and was without doubt topped with a dome, it could be concluded that the walls of the room 6 were most probably 5.7 meter high up to the bottom of the dome. The width of the room, counting from the western wall to the eastern one, was 5.7 meter. One other interesting fact about the room could be mentioned as it appears to be relevant for the understanding of the complex and the date of construction. The room no. 6 seems to have been a copy of the central space of the *triclinium* (dining room situated on the first floor of the Palace in its southeastern part) which was also of an octagonal shape with 4 niches. Thus, both rooms were products of the same architectural concept and were certainly built at the same time by the same craftsmen.

The same conclusion could be reached in the case of the room no. 4 which had a similar plan. Actually, its plan was nothing more than a small scale copy of the Palace's monumental vestibule. Thus, its original appearance can be deduced from the comparison with the vestibule which is still preserved up to the top of the dome. Furthermore, the same layout was attested in two rooms of the southwestern part of the palace which was usually perceived as the emperor's private chambers (Figure 1). Planimetric similarity between the vestibule and the room no. 4 incite further comparisons; one of them is based on the assumption that the room no. 4 had an elevation much alike the one of the vestibule. If this is correct, than the room no. 4 also had an *oculus* at top of the dome. Such an opening in the room 4, when combined with the fact that the room had a central plan and the apparent absence of a suitable space for the bath tub (it is more likely the room was equipped only with a *labrum* at the centre of the room), indicates that the room no. 4 must have been a second *sudatorium*, obviously colder than the one in room no. 6. Again, if we rely on Vitruvius, this room was 5 meter high up to the bottom of the dome, as the diameter of the room was 5 meter. In fact, there

are many reasons why we should rely on Vitruvius' recommendations in the case of Diocletian's palace. For example, the vestibule compared to room no. 6 perfectly embodied Vitruvius recommendations. Its height up to the bottom of the dome was about 11 meter, thus almost perfectly equaling its diameter, just as Vitruvius recommended. The height of the dome of the vestibule was about 5 meter. If we apply the same ratio between the height of the walls without the dome and height of the dome of the vestibule on the room no. 4 we can easily calculate that the room should have been 7.25 m high up to the top of the dome. Even more sizable was the room no. 6 which, if we use the same calculation, was 8.26 m high up to the *oculus* of the dome. These measures, which should be considered as more than probable, made these rooms quite remarkable and we can only wonder about their decoration.

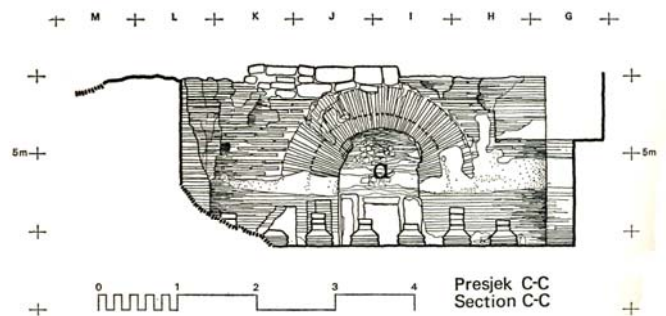


Figure 3 Section of the hypocaust chamber (room no. 5)

Judging by the size of apertures beneath the floor and the shape of the apartments, it is evident that the room 4 still belonged to the block of the hot rooms while the room no. 3 was a tepid one. Both rooms had hypocaust chambers which were fed by the hot air from the room no. 5, already changing its properties and becoming heavier. The question of the exact function of the doubled sweating room could be explained only by the fact that the bathers had to be gradually acclimatized to the temperature of the *caldarium*. After all, Vitruvius recommends exactly such a placement of the sweating room which, he wrote, should be adjacent to the tepid room [18]. At the same time, room no. 4 could have served as the *unctorium*. Discussion about the actual position of *unctorium*, the room in which the bathers applied the various oil-based preparations to their bodies, is still ongoing [19]. It is not yet clear if the position of the *unctorium* was conventionally fixed in the architecture of the baths. Anointment could have been performed in various stages of the bathing process depending on the preferences of the bather. Also, it should be bared in mind that "row type" baths were a two way structures, so some of the apartments could have been multifunctional when it comes to the bathing ritual. In entering the *caldarium*, room no. 4 could have served just as a *sudatorium*, while on the return it could have been used as an *unctorium*. Or other way around, according to the preferences of the bather.

However, it is worth noting that the rooms no. 4 and 6 had a more clearly pronounced socialization aspect, indicative of the capacity of the *balneum*. These two rooms, again judging

by the comparisons with the similar buildings and the sheer logic of structuring of space, were the place of tranquil relaxation and conversation. Four niches were most probably equipped with benches, each one large enough to accommodate one person. For example, the niches of the room no. 6 were 1.76 m² (with a diameter of 1.5 m). Their size implies that they were big enough for one person to lie down and relax on the bench. It is interesting that the niches in the room no. 4 were almost of the same size, although the room was quite smaller. Their diameter was just 10 cm smaller. Thus, we can deduce the capacity of this smallish *balneum* and we can assume the reasons why it fell into disuse in the following centuries. It was obviously too small to be of any use to the wider circle of residents in the Palace. Both of the rooms were evidently intended for four bathers sitting on the benches in the four niches, regulating the humidity of the room by pulling the chain holding the *clipeus* hanging beneath the *oculus*.

Thus, the hypocaust system and the reconstructed direction of the flow of hot air beneath the floors of the rooms 5-3 tell us much about the function of each of the rooms. In the western *balneum* of the Diocletian's palace we are confronted with an arrangement of specialized apartments more elaborate and certainly more luxurious, judging by the care with which the rooms were designed, than we find in the ordinary, even luxurious, private baths. Sequence *laconicum/caldarium-caldarium-sudatorium/unctorium-tepidarium* testifies that they were designed according to the specific preferences of the owner, in this case undoubtedly the emperor *Gaius Aurelius Valerius Diocletianus*.

However, returning to the technical solutions in the hypocaust, although the aperture ϵ was obviously the main element for the creation of draught in a line between rooms 5 and 3, the hypocaust system continued underneath the room no. 1 which was connected to the system only by a relatively small aperture. Comparison with various other baths shows that this room could have only been an *apodyterium* (dressing room). At the same time, the *apodyterium* was, without exception, an entrance room. Thus, the hypothesis that the western *balneum* of the Diocletian's palace had an entrance on the western side surely does not stand. This hypothesis is also put in question by the fact that the room 6a should be identified as a *praefurnium*. Although they did not explain, it seems that the researchers were misled by the two openings in the walls of the room 6a. However, these two openings were identified on the level of the floor or 10 cm above it (the floor which was actually some 2 meters below the floor of the room no. 6 because of the hypocaust). Thus, it is most probable that the opening connecting the rooms 6 and 6a was in fact an aperture of the furnace and not an entrance to the complex. There is not one example of baths with the entrance leading directly to the *sudatorium* or *caldarium*, as such ingress would have been extremely damaging for the health. It is more likely, although it was not confirmed by archaeological excavations, that the room no. 1 was actually the first room in a bathing sequence.

This conclusion is further strengthened by the fact that the room no. 1 was connected to the room no. 2 in which a round *piscina* (basin) was found. Unlike previously mentioned rooms, room no. 2 had no hypocaust and was sunken some 3

meters below the level of the other rooms. It was made clear by the last explorations that this room had a basin accompanied by a bench in the northern part of the room. Detachment of this room from the hypocaust system attested in all other rooms and the presence of a *piscina* and a bench above it, clearly show that the room served as a *frigidarium* (cold bath). Drainage channel found in 1972 in the room no. 7 could have been the one which led the used water from the *frigidarium* towards the sea. Its orientation and the avoidance of the main rooms certainly point that this could have been the case. It should be also noted that the *frigidarium* was the last stop for the water flowing into the basins and tubs of the *balneum*, so it is not surprising that it was situated at the lowest level of the complex, so the flow of the water could be secured.

One curious fact related to the room no. 2 is that the researchers from 1972 have documented the existence of a passageway leading south from the room. Passageway is documented as having stairs, so it could have lead only upwards into the imperial apartment. Unfortunately, the same researchers did not offer any comment on its existence nor did they provide any additional information about it, except those documented on their drawings. Although the passageway could have been a product of some later period, if it was in fact built at same time as the other parts of the *balneum*, its existence would provide a clear answer for whom the baths were intended for. However, we are certain that the passageway was not the main entrance to the complex and that the main entrance should be looked for at the northern side of room no. 1 which had all the properties of an *apodyterium*. So, the passageway could have only served as a back door of the complex enabling the entrance directly from the imperial apartments into the room with which the bathing sequence usually started and ended.

Rooms no. 7 and 8 have been somewhat better studied only in the most recent explorations, but the collated information is still inadequate to make any definite conclusion about their function or about the way they were connected to the hypocaust system. As mentioned above, room no. 7 was connected by a large arched opening ϵ with the hypocaust chamber beneath the room no. 3. So, it seems that the room also had a suspended floor although there is no mention of *pilae* in the reports from the archaeological explorations. Room no. 7 lead further to the barrel vaulted room no. 8 with a different orientation. Function of this room that had protruded into the back wall of the basements of the imperial apartment remains unknown.

Beside this description of the *balneum* one other issue should be mentioned as it is most relevant for the study of the heating system in this *balneum*. It is related to the finding of the pieces of *tubuli*, ceramic shafts designed and produced to serve as a part of the ventilation system built in to the walls of *balnea* and private homes. Findings of *tubuli* surely point to the fact that the walls of the *balneum* in Split were also heated, thus following the standard patterns of vertical distribution of heat. These findings are a perfect example of a need for interconnection of several methods in interpretation of Roman *balnea*. As demonstrated by several studies of the western baths in the Palace, archaeologists have been unable to determine the

importance of these finds. Moreover, they were puzzled by the fact that the *balneum* had walls made solely of bricks and not of stone, or a combination of stone and tiles (*opus mixtum*). However, pieces of *tubuli* found in the complex show that it most probably contained vertical heating elements which could have only been built into the brick walls and not into the ones made of stone.

Thus, after presenting physical facts defining the structure in question, as detailed as possible, we believe that a mathematical model could be applied to the heating system of this luxurious *balneum* in order to determine the properties and functions of its rooms.

MATHEMATICAL MODEL

Heating system of Roman *thermae* and *balnea*, in this particular case located in Diocletian's palace in Split, was based on a central heat production by firing wood or charcoal [20]. The *balneum* had originally at least 8 rooms, but archeological remains give relatively good insight into three centrally placed ones. The authors thus focused their attention solely to the thermodynamic calculation of the space heating system of these rooms, without depicting thermal treatment of water, what would be regarded as a relatively simpler task. The combustion chamber, called *prae-furnium*, was located underground with an open connection to ambient air. Flue gas was conducted to the hypocaust, a channel system of some 2 m height, with pillars for balneum floor support. The hypocaust was divided into three distinctive consecutive parts. From the first part, flue gas would flow to the second part, but beforehand a part of flue gas would be separated and led vertically through special hollow bricks, named *tubuli*, up to the collecting *tubuli* (mainly placed horizontally) at the vault or dome height, in order to outlet flue gas to the atmosphere. Not having enough data for precise calculation, it was assumed that 35 % of the flue gas would pass through the *tubuli* and the rest 65 % would proceed to the next room. The same method of heating was used in the second and third room.

In order to get an insight in heating efficiency and temperature distribution in the rooms of the described system, thermodynamic calculation has been conducted. Utilized fuel was charcoal, which releases some 28-30 MJ/kg of heat (H_d). In order to keep floor temperatures in the rooms at an acceptable level, the oxygen content in flue gas of 20.3 % was assumed, what gives the air access ration of 30, much higher than would be in modern heating systems. The specific flue gas volume V_G per kilogram of fuel than is calculated by firstly calculating theoretical and actual volume of air:

$$V_{Atheor} = \frac{1,867 \cdot r_c + 5,6 \cdot r_h - 0,7 \cdot r_o}{0,21} = 7.72 \quad \text{m}^3/\text{kg}_F$$

$$V_A = \lambda \cdot V_{Atheor} = 190.7 \quad \text{m}^3/\text{kg}_F$$

$$V_G = 1.867 \cdot r_c + 11.2 \cdot r_h + 1.24 \cdot r_w + 0.79 \cdot V_{Atheor} + 0.8 \cdot r_n + (\lambda - 1) \cdot V_{Atheor} = 190.9 \quad \text{m}^3/\text{kg}_F$$

With combustion efficiency, η_C , estimated at 95 %, and calculated lower heating value, H_d , of 29.76 MJ/kg (based on 80 % carbon content in charcoal), the theoretical temperature is 93.61 °C. Some of the important input data are given in table 1.

Table 1 Input data for thermodynamic calculation

Parameter	Symbol	Value	Unit
Fuel consumption	F	2	kg/h
Ambient temperature	T_0	0	°C
HTC of gas in hypocaust	α_H	22	W/m ² K
HTC of gas in tubuli	α_T	25	W/m ² K
HTC of air in room	α_R	7.7	W/m ² K
HTC of outside air	α_o	25	W/m ² K
Hypocaust conductivity	k_H	0.75	W/mK
Wall conductivity	k_w	1.00	W/mK

HTC heat transfer coefficient

Heat balance of a room encompasses the heat flow rate from the hypocaust (through the floor), Q_H , the heat flow rate from *tubuli* (through the wall), Q_T , and the rate of heat loss through the openings (doors, windows ...) as well as through the wall itself (between *tubuli*), Q_{RL} . Besides, there is a heat loss from *tubuli* to the outside. The index i denotes respective room.

$$Q_{H,i} + Q_{T,i} = Q_{RL,i}$$

The heat flow rate to the room from the hypocaust and flue gas enthalpy at the hypocaust exit are solved from the next two equations,

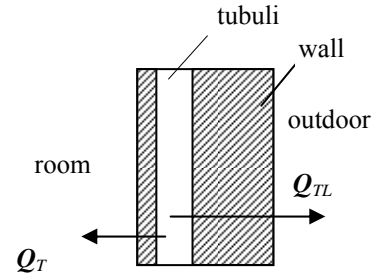


Figure 4 Wall cross section with *tubuli*

$$Q_{H,i} = \frac{1}{\frac{1}{\alpha_H} + \frac{s_H}{k_H} + \frac{1}{\alpha_R}} \cdot A_{H,i} \cdot \Delta T_{H,i}$$

$$Q_{H,i} = F \cdot (h_{G,i} - h_{G,i+1})$$

Theoretical enthalpy

$$h_{G,1} = \eta_C \cdot H_d$$

Heat flow rates are given by the next formulas

$$Q_{T,i} = \frac{1}{\frac{1}{\alpha_T} + \frac{s_T}{k_W} + \frac{1}{\alpha_R}} \cdot \xi_i \cdot A_{T,i} \cdot \Delta T_{T,i}$$

$$Q_{TL,i} = \frac{1}{\frac{1}{\alpha_T} + \frac{s_{TL}}{k_W} + \frac{1}{\alpha_0}} \cdot \xi_i \cdot A_{TL,i} \cdot \Delta T_{TL,i}$$

$$Q_{RL,i} = \frac{1}{\frac{1}{\alpha_0} + \frac{s_W}{k_W} + \frac{1}{\alpha_R}} \cdot (1 - \xi_i) \cdot A_{W,i} \cdot \Delta T_{R,i}$$

RESULTS

The described mathematical model is solved by the iterative technique on two levels. The upper level is solved for the room temperature. The lower level of iteration gives solutions for the flue gas enthalpy after the hypocaust and at the exit, what must be accomplished inside the upper level of iteration procedure.

The results are given in Table 2 for rooms no. 5 to 3. They are in a good agreement with Forbes [21], who stated that the temperatures in the hypocaust were expected to be in the range 60-80 °C. It was calculated with the assumption that air excess ratio was 30, while Forbes estimates were 10-15. With the lower air excess ratios, the floor temperatures would rise significantly.

Table 2 Calculation results ($\lambda=30$)

Parameter	Room5	Room4	Room3	Units
Heating rate	3.19	1.73	2.02	kW
FGT- hypocaust outlet	80.97	69.05	48.18	°C
FGT - therme outlet	47.12	47.59	38.31	°C
Floor temperature	48.21	43.76	29.76	°C
Room temperature	38.37	35.91	22.70	°C

FGT – flue gas temperature

Since the air excess ration has a great influence on the calculation results, the sensitivity analysis is carried out in the range of $\lambda=20-35$. Results are presented in the following tables.

Table 3 Room temperature (°C)

λ	Room5	Room4	Room3
35	34.17	32.25	21.08
30	38.37	35.91	22.70
25	43.65	40.35	24.37
20	50.48	45.66	25.76

Table 4 Hypocaust flue gas temperature at the exit(°C)

λ	Room5	Room4	Room3
35	71.07	61.97	45.33
30	80.97	69.05	48.18
25	93.91	77.61	50.76
20	111.23	87.76	52.14

Table 5 Floor temperature (°C)

λ	Room5	Room4	Room3
35	42.53	39.14	27.52
30	48.21	43.76	29.76
25	55.56	49.44	32.11
20	65.47	56.38	34.19

It could be argued that the floor temperatures above 50 °C would be difficult to stand, what was the main reason why the authors accepted the air access ratio of 30.

The heating efficiency is given in table 6 and figure 5. As the air access ratio is increasing the heat efficiency is going down, expectedly. Heating efficiency is defined as

$$\eta_{heating} = \frac{Q_{H,5} + Q_{T,5} + Q_{H,4} + Q_{T,4} + Q_{H,3} + Q_{T,3}}{F \cdot (H_d - h_{G,0})} \cdot 100\%$$

Table 6 Heating efficiency (%)

λ	Heating efficiency, %
35	37.91
30	41.96
25	46.81
20	52.54

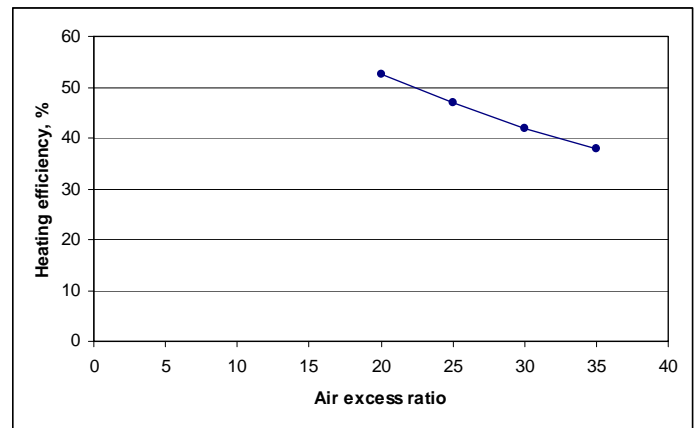


Figure 5 Heating efficiency as a function of the air excess ratio

Heating efficiency is relatively low compared to contemporary systems, mainly due to the high air excess ratio, which increases flue gas exit loss.

NOMENCLATURE

A	m^2	surface area
F	kg/s	fuel mass flow rate
H_d	kJ/kg	lower heating value
h_G	kJ/kg _F	flue gas enthalpy per fuel kilogram
k	kW/mK	heat conductivity
Q	kW	heat flow rate
r_c		mass content of carbon in fuel
r_h		mass content of hydroge in fuel
r_n		mass content of nitrogen in fuel
r_o		mass content of oxygen in fuel
r_w		mass content of water in fuel
s_H	m	floor thickness
s_W	m	wall thickness
T	°C	temperature
V_A	m^3/kg_F	air volume per fuel kilogram
V_{Atheor}	m^3/kg_F	theoretical air volume per fuel kilogram
V_G	m^3/kg_F	flue gas volume per fuel kilogram
ΔT	°C	average logarithmic temperature difference
λ		air excess ratio
α	kW/m ² K	heat transfer coefficient
η_c	%	combustion efficiency
ξ		portion of wall area covered by tubuli

Index

A	air
C	combustion
G	flue gas
H	hypocaust
HL	heat loss
i	room number
R	room
RL	room (heat) loss
T	tubuli
TL	tubuli (heat) loss
O	ambient

CONCLUSION

In all, the western *balneum* with its elaborate overall plan, its sizable rooms some of which had, namely the room no. 6, quite a imaginative design, repetition of the shapes and forms which we meet in the most representative part of the palace as well as in the one which might have served as the emperor's personal chambers, the usage of the luxurious materials as for example porphyry marble found in the drainage channel in the room no. 7 [13], was without doubt an imperial structure whose splendor and elaboration obviously surpassed the majority of private baths of the time [22]. In the architectural sense, they were quite innovative and a product of a skilled architect working with same craftsmen engaged on the other parts of the imperial palace. Technologically, these baths also represent an extraordinary case since they had a hypocaust system whose height by far exceeds the one that we meet in non-imperial *balnea* or even in public baths. Knowing that the emperor Diocletian was a person of frail health in his later age and one with a very sophisticated architectural taste we wonder what could have these western baths offered him.

It is a fact that often by means of archaeological excavations, interpretations of the findings, as well by the art historians' morphological analysis, the function of every single space / room of a roman bathing complex cannot be determined without reasonable doubt. There are some rules indicating the most probable position for every function, as has been depicted, but no strict ones. As was shown, the mathematical model for temperature distribution clearly points to the functions of every single space, thus showing the efficiency of such interdisciplinary work. It was also shown that the air excess ratios in Roman *paefurnium* had been of much higher values than was believed so far. As for the temperature distribution in this case study, it is consistent with the theoretical and experimental attempts conducted till now, thus showing the efficiency of our mathematical model.

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