Learning to Learn: An Epistemological Paradox in Education

Absence of finality is of the essence of the scientific spirit.


The epistemological positions strongly influence the views of education. By accepting a specific notion of knowledge we seem to be committed to specific educational aims. Two notions of knowledge can be easily extracted. The notion of knowledge as a state arises out of the conviction that knowledge can reach its final state. The opposite belief motivates notion of knowledge as a process. The two notions should not be confused with the distinction between a process and its result. The terms ‘knowledge-state’ and ‘knowledge-process’ have mutually exclusive meaning. If one thinks that our beliefs can be crystallised into an undeniable form, then she accepts the notion of knowledge-state. On the other side, if one considers belief systems as subject to constant revision, then she accepts the notion of knowledge as an open-ended process.

Let us assume that knowledge-state is a philosophically sound notion of knowledge. Next, let us assume that the final state of a body of knowledge is neither innate nor revealed to a knowing agent. From these assumptions it follows that a knowing agent (or a group of knowing agents) must acquire knowledge in the process learning by exploring. There are no reasons to prevent the transmission of knowledge in a process that is not similar to the original process of its acquisition. If a piece of final knowledge has a value, then its adoption has a value, too. Therefore, the educational ideal arising from the notion of knowledge-state is the ideal of learned man. Within this framework the cognitive aspect of the problem of education is reduced to the problem of parsimonious and efficient mode of learning by being told by someone who has already mastered the subject matter.

Let us make a contradictory assumption: it is the notion of knowledge-process that is philosophically warranted. If any resulting state of a cognitive process is only a passing stage in an open-ended process, then the mastering of a content does not seem a reasonable choice for the role of the highest educational aim within the cognitive domain. The value of any stage of knowledge-process is of lesser degree than the value of a higher stage (although not necessarily lesser than the value of any subsequent stage). In this dynamic conceptual network the educational ideal is connected with the ability of surpassing the attained stage, and therefore its marks are critical and explorative attitude. The ultimate end of cognitive education is to be sought in developing those skills that enable acquisition of a new knowledge. Learning skills as any other skills can be improved only through exercise i.e. by learning. Looking downward through the hierarchy of educational aims, even the lower positioned aims can not defined exclusively in terms of understanding of a given body of
knowledge, rather they include an understanding of the nature of knowledge\(^1\). By assuming that the notion of knowledge process is more realistic than the notion of knowledge-state, we seem to be forced to the conclusion that one-sided concentration on efficient transmission of the current knowledge-state might produce undesirable effect of decrease of human adaptive capabilities.

The notion of knowledge-process is deeply rooted in the philosophical thought and it can be traced back to Socrates’ insights into imperfection of the human knowledge and its dialogical construction. Twentieth century thinkers’ theories converged towards a dynamic notion of knowledge: incompleteness of the human knowledge is not its temporary weakness, rather, it is its inherent property. The notion of knowledge-process has been delineated from diverse perspectives. A brief summary of some influential perspectives could elucidate the notion of knowledge-process. John Dewey, one of the most prominent philosophers of education, exposed a system of instrumentalist logic in *How We Think*, the work devoted to educators. The initial step in cognition i.e. inquiry is triggered by the doubt and uncertainty experienced when our action is disrupted. The knowledge is a solution for the problematic situation, but the fact of constant change of circumstances can make it useless to adopt an old solution for a novel problematic situation. In Jean Piaget’s theory of genetic epistemology the constructive nature of knowledge has been emphasised. The knowledge is an adaptive process having two principal forms; in assimilation an already existent cognitive structure incorporates new information or provides a solution for a practical problem, whereas in accommodation a cognitive structure is being transformed into a new one. The cognizability is a relative notion; an unsolvable problem may become solvable if the cognitive structure gets transformed to a higher level. Kurt Gödel has proved the incompleteness theorem for the deductive theories formulated using languages rich enough to express their own syntax. The inevitable existence of undecidable sentences implies impossibility of giving the consistency proof. Therefore, the deductive theories can not be closed within themselves, for consistency proof a richer theory is required, and for it the same arguments apply making thus an infinite procedure out of the construction of a deductive theory. According to the philosophical thesis attributed both to Alonzo Church and to Alan Turing, the intuitive notion of algorithmically solvable problem can be given a precise mathematical definition\(^2\). There are problems that can not be solved by an algorithm (e.g. Turing’s halting problem). In his rational reconstruction of the history of ideas Karl Popper has relied on the simple but usually overlooked fact that no number of observations can verify a general, either deterministic or probabilistic, statement covering an infinite domain of cases. The impossibility of verification implies the possibility

---

1 A fully developed learning ability implies reflection on one’s one cognitive process and some kind of control over it. These skills are usually called metacognitive skills and their mastering involves understanding of the nature of the knowledge. It follows that adopting of a kind of metatheory is required for the development of metacognitive skills. “My thesis is that education ought to be the transmission of understanding-knowledge and that an educated person will properly be one who not only has understanding-knowledge but understands the nature of that knowledge.” Cooper, N. Transmission of Knowledge. in *Philosophers on Education*, edited by R. Staughton and J. Wilson. Totowa, New Jersey: Barnes and Noble, 1987. pp. 61-78.

2 “Which numerical functions f: N\(^n\)→N are computable? (…) this question had some earlier answers, of which I shall single out the following three: 1. f is recursive (Gödel, Kleene), 2. f is computable on an abstract machine (Turing, Post). 3. f is definable in the untyped \(\lambda\)-calculus (Church, Kleene). These tentative answers were shown to be equivalent by Church (1936.) and Turing (1936-7.).” in Lambek, J. Programs, grammars and arguments: a personal view of some connections between computation, language and logic, *The Bulletin of Symbolic Logic*, vol.3. no.3. Sept.1997. pp.312-313
of refutation for any empirical theory. Thomas Kuhn’s postpositivistic history of science has revealed a conceptual discontinuity in succession of paradigms. Jean Francois Lyotard’s analyses have shown the impossibility of foundational and unifying metanarratives. This brief survey of some epistemological or epistemologically relevant thoughts shows that dynamic notion of knowledge has its strong philosophical background.

Summarising the theses of the above-mentioned authors we may concede that knowledge can not be completed, or that the notion of knowledge-state is acceptable as a descriptive concept and it is unacceptable normative concept. The education theoretic reaction to the insights into inevitable limitations imposed on human knowledge has been manifested in a dynamic concept of educational aims: the real substance lies in learning how (to learn) rather than in learning that. The educational philosophy of constructivism is usually understood as being motivated by metatheoretical insights of the kind mentioned above.

Four questions are distinguishable in the problem space of educational choice for cognitive domain: 1. Who is being taught? 2. What to teach her? 3. How to teach? 4. Why to teach? In this complex net of pedagogical choices the fixed points are given by opting for a theory of learning (Who?) and by opting for an educational aim (Why?), the other two are dependent (What? How?). The choice of educational aim for cognitive domain can be conceived as localisation of a point within a spectrum defined by two extreme points standing for acquisition of a specific knowledge and for development of acquiring knowledge skill, respectively. Extreme positions are theoretical posits: the skill of knowledge acquisition can not be developed in an empty space without a specific knowledge that is being acquired, on the other hand, no knowledge can be acquired without a skill to acquire it. The independence of educational aim is a relative matter: the choice of an educational aim becomes a dependent choice when put in wider philosophical setting. The epistemological insight into dynamic nature of knowledge motivates a shift towards skills part of the spectrum. Consequently, the educational aims for the cognitive domain are redefined in terms of intellectual abilities, skills and attitudes. Some examples taken from literature on educational psychology and didactics can illustrate the point. Not so long ago, Benjamin Bloom’s taxonomy of educational objectives was an exemplary way of approaching the problem: the hierarchy of categories of educational objectives for the cognitive domain was defined by recurrence to the mode of mastering a subject matter. Twenty years later, Lauren Resnick defines educational aims exclusively in terms of thinking skills where the uppermost position belongs to higher-order

---

3 “Put into simple terms, constructivism can be described as essentially a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive acts.” M.R. Matthews. Old wine in new bottles: a problem with constructivist epistemology. *Philosophy of Education Society Yearbook* 1996. University of Illinois. In historical perspective, the idea of knowledge as construction can be attributed to Giambattista Vico (De antiquissima Italorum sapientia, 1710. ).


5 Bloom’s ascending hierarchy of cognitive categories is given by 1. knowledge, 2. comprehension, 3. application, 4. analysis, 5. synthesis, 6. evaluation. The instructional objectives are defined as skills (examples for derived objectives: 1. knows common terms, 2. interprets verbal material, 3. applies principles to new situations, 4. recognises unstated assumptions, 5. writes a well-organised theme, 6. judges the consistency of written material), still the mode of mastering a given knowledge dominates in Bloom’s conceptualisation of educational aims.

thinking\textsuperscript{7}. The multitude of interpretational perspectives and readiness to revision characterise higher order thinking, which can not be defined in the terms of subject matter because it is yet to be established by a cognitive process of the kind. In the constructivistic approach to teaching\textsuperscript{8} learning is the product of self-organisation. The teacher’s role is redefined: the teacher is not the one who has the possession of privileged truths, rather the teacher is an explorer trying to build a viable model of a student’s conceptual network in order to create a learning context that prompts the process of self-regulation.

Epistemological considerations influence our choice of cognitive educational aim. If the knowledge were a possession of a finished product not subject to changes then we would have a reason to seek for the balance between the development of skill and the adoption of knowledge. If knowledge is not a finished product, and if furthermore can not be finished at all, then adoption of revisable and incompletable knowledge looses its attractiveness, while skill development comes to the fore. By placing the educational aim for the cognitive domain nearer to the ‘cognitive skills end’, opting for specific teaching methods becomes more important than choosing the specific subject matter. The means for end thus conceived is to be sought for primary in the field of teaching methods, with curriculum options receiving second order instrumental status. Skill develops through exercise. In order to develop the skill for knowledge acquisition one must engage in the process of learning. The logical priority belongs to ‘learning by exploring’, and not to ‘learning by being told’. ‘Learning by being told’ presupposes that something has already been ‘learned by exploring and discovery’. Therefore, the “teaching method” that must be used in purpose to develop cognitive skill must resemble real process of acquiring knowledge.

It is necessary to assess the realizability of an educational aim for an educational philosophy. It is an intuitive principle of practical rationality that an impossible goal is to be given up. The process of learning must be explored and analysed in order to find the answer to the question \textit{Is learning learnable?} For that purpose, a Popperian model of knowledge will be utilised. In a simple model, the process of learning by exploring is decomposed in following stages: a hypothesis formulation, deduction of a prediction, addition of an observational statement, elimination of the falsified prediction and refutation of the hypothesis\textsuperscript{9}. Those stages form a kind of a spiral with possible repetitions of the whole sequence and with iterations of the predictive sub-sequence. Popperian disregard for induction is acceptable to a degree: indeed, there is no induction starting from the theoretical neutral description of the reality. But there are, nevertheless, specific cognitive processes involved in hypothesis formation (induction) and hypothesis selection (abduction) which are not deductive in nature. The logical priority in knowledge acquisition belongs to formation of

\begin{footnotesize}
\textsuperscript{7} Higher-order thinking is nonalgorithmic, complex, yields multiple solutions, involves nuanced judgement and interpretation, involves the application of multiple criteria, involves uncertainty, involves self-regulation of the thinking process, involves imposing meaning, it is effortful.


\textsuperscript{9} The effect of addition of an observational statement that contradicts deduced prediction can be formally represented as follows. Let T be a consistent empirical theory and H the only hypothesis used in deduction of a falsified prediction p. The result of revision of T by \( \neg p \) is denoted by \( T \ominus \neg p \), which can be defined as \( Cn(T \uplus p) \cup \{\neg p\} \), where \( Cn \) is given consequence operation and \( T \uplus p \) denotes contraction by p (i.e. maximal subset of T that fails to imply p). It is obvious that \( H \notin T \ominus \neg p \). See Alchourrón, C.E., Gärdenfors, P., Makinson, D. On the logic of theory change: partial meet contraction and revision functions. \textit{The Journal of Symbolic Logic}. vol. 50, no. 2, 1985.
\end{footnotesize}
hypothesis. The deductive step follows, issuing usually in a singular statement that should be checked against empirical evidence. If the hypothesis is not confirmed it should be refuted, else deduction step is to be repeated. This repetitive sequence should not to be conceived as endless loop, although no number of confirmations suffice for the truth of a hypothesis covering an unlimited number of instances. In historical perspective, the case of falsification is a common outcome. In that case the hypothesis should be refuted. The refutation may imply an extensive reconstruction of a belief system, since at least one more general hypothesis entailing falsified one should be refuted. It seems that in this case a rationality principle is employed favouring revision that allows for the exceptions over strict refutation. Still, the refutation and the consequent reconstruction by formation of a new hypothesis are epistemically more important outcomes since they may result in an increase in predictive and explanatory power. The notion of learning as knowledge acquisition inherits a normative element inherent in the notion of knowledge. Leaving philosophical disputes aside; we may take predictive and explanatory power as the measure of epistemological adequacy. Therefore in a broad perspective, the stage of hypothesis formation has the central position in the learning process. A distinction must be made between abduction and hypothesis generation. Although in both cases an agent is involved in a process of ‘looking for an explanation’ there is an important difference in the way relevance of a hypothesis is dependent on previous knowledge. Abduction is process of selection where the hypotheses are already given. An agent has at her disposal a set of nomic statements. For example, such a statement could have the form ‘If an event of the type C occurs then (in a specified interval) an event of type E occurs’ or ‘If an object has property F, then it has property G’. Having encountered an event of the type E we may look for a correspondent nomic statement in order to explain its occurrence, and having encountered (or having caused) an event of the type C we may make a prediction. If there is more than one nomic statement mentioning events of the type E (or of the type C) the agent must select a hypothesis as the deductive basis of explanation (or prediction). The relevance of abducted hypothesis can not exceed the relevance of any hypothesis from the set. The process of a new hypothesis formulation is a more complex one since there are no hypotheses to choose from. An inductive algorithm may produce a hypothesis. The dependence of hypothesis relevance on the relevance of properties included in the description space is a very important fact. The relevance of algorithmically generated hypothesis depends on the relevance of the background knowledge. The effectiveness and soundness of any inductive algorithm depends on the relevance of the properties defining description space. The problem of reconstructing description space is algorithmically unsolvable. Random choice, heuristic strategies and creativity are needed for exceeding the relevance of prior (background) knowledge since the relevance of algorithmically generated hypothesis depends on the relevance of attributes selected. Therefore, the crucial form of knowledge acquisition\(^\text{10}\) (as an autonomous and a relevance upgrading process) can not be

\(^{10}\) Similar argument can be developed from the standpoint of formal learning theory. Usual paradigm tries to give a formal reconstruction for following concepts: (i) a theoretically possible reality, (ii) an intelligible hypothesis about reality, (iii) the data available about any given reality, were it actual, (iv) a scientist (or child), (v) a successful behaviour by a scientist working in a given, possible reality. In that framework a scientist (or a child) is identified with a function \(\psi\) from set \(\text{SEQ}\) of sentences in given language \(L\) (natural language or language of nature) to a hypothesis \(N\) (about grammar or laws). See Osherson,D., Scott,W., de Jongh,D., Martin,E. \textit{Formal Learning Theory}. ILLC Research Report and Technical Notes Series, Amsterdam, 1994. The following proposition can be proved: Let \(S\) be any countable collection of functions from \(\text{SEQ}\) to \(N\) (conceived as scientists). Then there is an identifiable collection \(L\) of languages such that no member of \(S\) identifies \(L\). We can give the interpretation for the proposition along the following lines: there is a class of learning problems that can not be solved by ‘computable functions’, where ‘computable function’ stands for function computable on a theoretical machine (‘Turing machine’) or ‘clerical procedure’ (Jeffrey) or ‘inductive algorithm’. The proof rests on lemma that there are uncountable many possible hypotheses and only countable many Turing machines.
reduced to ‘clerical procedure’ (‘Turing’s machine’) unless we are willing to accept *regressus in infinitum*. Still, there are some heuristic strategies that can be employed. An important strategy uses metaphor: find the familiar structure that is similar to certain degree to the one being explored and look for structural similarities in order to refine description space.

In educational setting the situation in which ‘a student has what a teacher does not have’ (a misconception) opens up the possibility for the sequence ‘hypothesis – false prediction’. The situation in which ‘a teacher has what a student does not have’ (missing conception) opens up the possibility for ‘hypothesis formation’. Pedagogical problem is communicative: it is students belief system that must be discovered. A counterexample can trigger learning process (process of cognitive self-regulation11), but it can not be given unless a portion of student’s theoretical model has been discovered.

It is possible to engage the other into knowledge-process, but it is impossible to have control over it. Inductive algorithms can be learned but they are not fully reliable12 and they depend on the previous knowledge. A creative act in cognition must be hypothesised in order to explain the knowledge relevance upgrade. The creativity by definition is not a ‘clerical procedure’. It is not learnable. Therefore, if learning is defined as a process of knowledge acquisition in which there is an increase in predictive and explanatory power, then it is not learnable how to learn. On the other hand, the development of cognitive skills is epistemically warranted educational aim. So it seems that there is still another dilemma in education: whether to choose attainable but inauthentic aim of mastering a knowledge-state or to choose authentic aim, ‘learning to learn’ for which there is no efficient procedure. The solution may lie in the direction of Rousseau’s concept of negative education: education is an art of avoiding harmful influences. Surely, it can not be harmful to provide a creative learning context for a student’s learning ability and to expose her to counterexamples. On the other hand, it may be harmful to adhere to the automation principle according to which the only worthwhile knowledge is the knowledge that can be taught to a computer, the procedural knowledge that can be decomposed into an algorithm.

**Antinomies of education** have attracted a lot of attention in philosophy. Patrick Suppes13, for one, has listed four antinomies: antinomy of adjustment (adjustment or

---

11 «In one of his most famous experiments, Piaget asked children, ”What makes the wind?” A typical Piaget dialogue: Piaget: What makes the wind? Julia: The trees. P: How do you know? J: I saw them waving their arms. P: How does that make the wind? J (waving her hand in front of his face): Like this. Only they are bigger. And there are lots of trees. P: What makes the wind on the ocean? J: It blows there from the land. No. It's the waves « Seymour Papert in the article on Piaget written on the occasion *Time* nomination of the greatest thinkers and scientists in the 20th century. ([http://www.time.com/time/time100/scientist/](http://www.time.com/time/time100/scientist/)) The example readily lends itself to analysis: Piaget’s counterexample (wind blows at sea and there are no trees there) causes a process of self-regulation. Julia’s cognitive reaction proceeds in two steps: first she constructs a specialization on the original hypothesis, and than she constructs the hypothesis that is on the same level of generality. The second construction is recognized as more stable by the subject’s reflection (probably because it follows from the deeply rooted and kinaesthetically confirmed hypothesis on moving objects causing flow of air).


achievement?), antinomy of method (content or method?), antinomy of child (child or curriculum?), antinomy of freedom (freedom or discipline?). It seems that here discussed epistemological paradox can be added to the list: unlearnable learning to learn or learnable learning to know. Education seems to be the area of inevitable antinomies. Therefore, teachers’ education should be philosophical in nature in order to enable reflective practice. Theachers’ reflective practice encompasses the awareness of complexity and uncertainty of pedagogical choices\textsuperscript{14}. On the other side, the shift towards cognitive skills as genuine educational aims strongly emphasises the need of the inclusion of philosophy in an authentic education\textsuperscript{15}.

Abstract

In the article the author explores influence of views of knowledge on the choice of educational aims. Having drawn the distinction between knowledge-state and knowledge process, the author shows that relevant philosophical and meta-theoretical results have given support to the notion of knowledge-process. Educational impact of that notion has been shown in defining of the educational aims in terms of skills development. The ability to learn is recognized as central skill. Examination of knowledge process shows that upgrading the relevance of the knowledge requires a step that cannot be decomposed into an algorithm. The antinomy arises: it is philosophically sound to choose the development of the learning ability as the central goal of the intellectual education, and there are no effective means for achieving the goal. Stronger inclusion of philosophical education has been advocated: it is needed both in education directed towards the development of the cognitive and meta-cognitive skills, and in professional preparation of the teachers.

\textsuperscript{14} “Clearly issues about how we teach – about pedagogy, management, resources, and so on – must depend crucially upon our stance towards such [philosophical] questions. This is not, of course, to commit the old foundationalist error of believing that we need to have settled such questions before we can start to hone our techniques – for there is a clear sense in which they are not finally resolvable; the point is rather that one is unlikely to flourish as an agent of education unless genuine engagement with them crucially informs one’s practice.” Carr, D. Toward a Re-Evaluation of the Role of Educational Epistemology in the Professional Education of Teachers. Philosophy of Education Society Yearbook 1998.

\textsuperscript{15} “One might say that whereas subjects like science, history and mathematics call upon skills and strategies associated with good reasoning and inquiry, concept formation and the like, it is philosophy which may be said to teach them” Splitter, L.J. On the theme of “Teaching for Higher Order Thinking Skills”. Inquiry . vol.XIV, no.4. 1995.