ELECTROMAGNETIC FIELD COUPLING TO ARBITRARY WIRE CONFIGURATIONS ABOVE A LOSSY GROUND USING THE ANTENNA THEORY APPROACH

D. Poljak[1], V. Doric[1], Khalil El Khamlichi Drissi[2]

[1]University of Split, FESB, R.Boskovica 32, HR-21000 Split, Croatia, dpoljak@fesb.hr
[2]Blaise Pascal University, Clermont-Ferrand, Pascal Institute, France, drissi@lasmea.univ-bpclermont.fr

Abstract. The paper deals with the electromagnetic field coupling to multiple wires of arbitrary shape above a lossy ground using the antenna theory approach. The problem is formulated in terms of the set of Pocklington integro-differential equations which is solved via the Galerkin-Bubnov scheme of the Boundary Element method (GB-IBEM). Some illustrative computational examples related to power line communications (PLC) systems are given in the paper.

I. INTRODUCTION

The electromagnetic field coupling to arbitrary configuration of aboveground wires has many applications in various aspects of EMC [1]-[9], and can be analyzed by using the simplified transmission line model or rigorous, but computationally more demanding antenna theory model in either the frequency or time domain, respectively [1]-[3].

In particular, if curved wires are of interest the use of antenna theory approach appear to be necessary for the satisfactory level of accuracy [1].

Electromagnetic fields illuminating the aboveground wire structure induces the current to flow along the wires. These currents also generate scattered fields propagating away from the structure.

The present work reviews the analysis of arbitrarily shaped wires radiating over a lossy half-space by means of the antenna theory approach.

The formulation is based on the set of Pocklington integro-differential equations for curved wires. The presence of a lossy medium is taken into account by means of the Fresnel plane wave reflection coefficient (RC) for TM and TE polarization, respectively [1]. The corresponding numerical solution is carried out via the Galerkin-Bubnov variant of the Boundary Element method (GB-IBEM). Some illustrative computational examples pertaining to power line communication (PLC) systems are given in the paper.

II. FORMULATION

The geometry of interest is related to an arbitrary overhead wire configuration, as shown in Fig 1. The set of coupled Pocklington-integro-differential equations for arbitrarily shaped wires can be readily obtained as an extension of the single wire integro-differential equation. The single wire Pocklington equation can be derived by featuring the continuity condition for the tangential components of the electric field at the wire surface:

$$E_s^{inc} + E_s^{sct} = Z_L^{'} I(s) \tag{1}$$

where , *I(s')* is the unknown current distribution along the wire, $\vec{E}^{exc}$ stands for the excitation field, $\vec{E}^{sct}$ denotes the scattered field and $Z_L^{'}$ is the corresponding conductor per length impedance of the conductor.

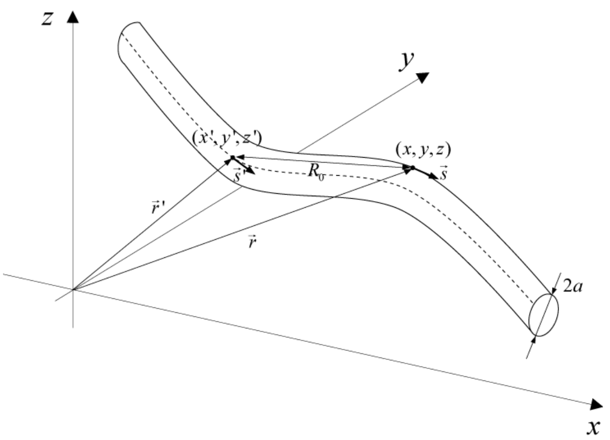

Fig. 1. Wire configuration of arbitrary shape

Performing certain mathematical manipulation yields the Pocklington equation for the imperfectly conducting wire or the wire containing the load impedance

$$\begin{aligned} E^{inc}(s) = -\frac{1}{j4\pi\omega\varepsilon_0}\int_0^L \Bigg\{ \left[k_1^2 \vec{e}_s \vec{e}_{s'} - \frac{\partial^2}{\partial s \partial s'} \right] g_0(s,s') + \\ + R_{TM} \left[k_1^2 \vec{e}_s \vec{e}_{s*} - \frac{\partial^2}{\partial s \partial s*} \right] g_i(s,s*) \Bigg\} I(s')ds' + Z_L^{'} I(s) \end{aligned} \tag{2}$$

where $\vec{e}_p$ is the unit vector normal to the incident plane, $g_0(s,s`)$ denotes the unbounded lossless medium Green function:

$$g_0(s,s') = \frac{e^{-jkR}}{R} \tag{3}$$

and R is the distance from the source point to the observation point, respectively. The propagation constant of free space is:

$$k^2 = \omega^2 \mu_0 \varepsilon_0 \tag{4}$$

Furthermore, $g_i(s,s^*)$ arises from the image theory and is given by:

$$g_i(s,s^*) = \frac{e^{-jkR^*}}{R^*} \tag{5}$$

and R^* is the distance from the image source point to the observation point, respectively.
The influence of a lossy half-space is taken into account via the Fresnel plane wave reflection coefficient (RC) for TM and TE polarization, respectively [1]:

$$R'_{TM} = \frac{\underline{n}\cos\theta' - \sqrt{\underline{n} - \sin^2\theta'}}{\underline{n}\cos\theta' + \sqrt{\underline{n} - \sin^2\theta'}} \ , \ R_{TE} = \frac{\cos\theta' - \sqrt{\underline{n} - \sin^2\theta'}}{\cos\theta' + \sqrt{\underline{n} - \sin^2\theta'}} \tag{6}$$

where θ' is the angle of incidence and $\underline{n}$ is given by:

$$\underline{n} = \frac{\varepsilon_{eff}}{\varepsilon_0}, \quad \varepsilon_{eff} = \varepsilon_r \varepsilon_0 - j\frac{\sigma}{\omega} \tag{7}$$

and ε_{eff} is the complex permittivity of the ground.

An extension of the single wire case leads to the set of the Pocklington integro-differential equation for a multiple wires of arbitrary shape above a lossy half-space is given by

$$\begin{aligned} E_{sm}^{exc}(s) = {} & \frac{j}{4\pi\omega\varepsilon_0} \sum_{n=1}^{N_w} \int_0^{L_n} \left\{ \left[k^2 \vec{e}_{s_m} \vec{e}_{s_n'} - \frac{\partial^2}{\partial s_m \partial s_n'} \right] g_{0n}(s_m, s_n') + \right. \\ & + R_{TM} \left[k^2 \vec{e}_{s_m} \vec{e}_{s_n^*} - \frac{\partial^2}{\partial s_m \partial s_n^*} \right] g_{in}(s_m, s_n^*) + \left(R_{TE} - R_{TM} \right) \vec{e}_{s_m} \vec{e}_p \cdot \\ & \left. \cdot \left[k^2 \vec{e}_p \vec{e}_{s^*} - \frac{\partial^2}{\partial p \partial s^*} \right] g_i(s_m, s_n^*) \right\} I(s_n') ds' \end{aligned} \tag{8}$$

where $\vec{E}_{sm}^{exc}$ is the excitation field, N_w is the total number of wires, $I_n(s_n')$ is the unknown current distribution induced on the n-th wire.
The Green functions $g_{0mn}(x,x`)$ and $g_{imn}(s,s`)$ are of the form:

$$g_{0mn}(s_m, s_n') = \frac{e^{-jk\,R_{1mn}}}{R_{1mn}}, \quad g_{imn}(s_m, s_n') = \frac{e^{-jk\,R_{2mn}}}{R_{2mn}} \tag{9}$$

Once the current distribution is determined the electric field due multiple wires of arbitrary shape can be computed as reported in [1].

III. NUMERICAL SOLUTION

The set of Pocklington integro-differential equations (8) is numerically solved via the Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM). An outline of the method is given here, for completeness, while the full mathematical description of the method could be found elsewhere, e.g. in [3].
Performing the Galerkin-Bubnov scheme of (GB-IBEM) the set of coupled integro-differential equations (8) is transformed into the following matrix equation [1]:

$$\sum_{n=1}^{M} \sum_{i=1}^{N_n} [Z]_{ji}^{e} \{I\}_i^{e} = \{V\}_j^{e} \tag{10}$$

where the mutual impedance matrix is given by [1]:

$$\begin{aligned} [Z]_{ij}^{e} = {} & -\int_{-1}^{1}\int_{-1}^{1} \{D\}_j \{D'\}_i^T g_{0nm}(s_n, s_m') \frac{ds_m'}{d\xi'} d\xi' \frac{ds_n}{d\xi} d\xi + \\ & + k_1^2 \vec{e}_{s_n} \vec{e}_{s_m} \int_{-1}^{1}\int_{-1}^{1} \{f\}_j \{f'\}_i^T g_{0nm}(s_n, s_m') \frac{ds_m'}{d\xi'} d\xi' \frac{ds_n}{d\xi} d\xi - \\ & - R_{TM} \int_{-1}^{1}\int_{-1}^{1} \{D\}_j \{D'\}_i^T g_{inm}(s_n, s_m^*) \frac{ds_m'}{d\xi'} d\xi' \frac{ds_n}{d\xi} d\xi + \\ & + R_{TM} k_1^2 \vec{e}_{s_n} \vec{e}_{s_m^*} \int_{-1}^{1}\int_{-1}^{1} \{f\}_j \{f'\}_i^T g_{inm}(s_n, s_m^*) \frac{ds_m'}{d\xi'} d\xi' \frac{ds_n}{d\xi} d\xi \\ & + \frac{j}{4\pi\omega\varepsilon_0} \int_{-1}^{1} Z_L' \{f\}_j \{f'\}_j^T \frac{ds_n}{d\xi} d\xi \end{aligned} \tag{11}$$

while the voltage vector is given by [1]:

$$\{V\}_j^n = -j4\pi\omega\varepsilon_0 \int_{-1}^{1} E_{s_n}^{exc}(s_n) f_{jn}(s_n) \frac{ds_n}{d\xi} d\xi_n \tag{12}$$

Once evaluating the current distribution along the wire the related radiated field can be obtained applying the certain BEM formalism [1].
The total field can be written, as follows:

$$\vec{E} = \sum_{k=1}^{N} \left[\vec{E}_{Sk}^e + R_{TM} \vec{E}_{Ik}^e + \left(R_{TE} - R_{TM} \right) \left(\vec{E}_{Ik}^e \cdot \vec{e}_p \right) \vec{e}_p \right] \tag{13}$$

where the field components due to radiation from a wire segment radiation are given by:

$$\vec{E}_{Sk}^e = \frac{1}{j4\pi\omega\varepsilon_0} \sum_{i=1}^{n} \left[\begin{aligned} & k^2 \int_{-1}^{1} \vec{e}_{ks} I_{ik}^e f_i(\xi) g_{0k}(\vec{r}, \vec{r}') \frac{ds_k'}{d\xi} d\xi \\ & + \int_{-1}^{1} I_{ik}^e \frac{\partial f_i(\xi)}{\partial \xi} \nabla g_{0k}(\vec{r}, \vec{r}') \frac{ds_k'}{d\xi} d\xi \end{aligned} \right] \tag{14}$$

$$\vec{E}_I^e = \frac{1}{j4\pi\omega\varepsilon_0}\sum_{i=1}^{n}\left[k^2\int_{-1}^{1}\vec{e}_{ks*} I_{ik}^e f_i(\xi) g_{ik}(\vec{r},\vec{r}*)\frac{ds_k'}{d\xi}d\xi \right. \quad (15)$$

$$\left. -\int_{-1}^{1} I_{ik}^e \frac{\partial f_i(\xi)}{\partial \xi'}\nabla g_{ik}(\vec{r},\vec{r}*)\frac{ds_k'}{d\xi}d\xi \right]$$

The total magnetic field is given by [1]

$$\vec{H} = \sum_{k=1}^{N}\left[\vec{H}_{Sk}^e + R_{TE}\vec{H}_{Ik}^e + \left(R_{TM} - R_{TE}\right)\left(\vec{H}_{Ik}^e \cdot \vec{e}_p\right)\vec{e}_p\right] \quad (16)$$

while the magnetic field components are given by [1]:

$$\vec{H}_{Sk}^e = -\frac{1}{4\pi}\sum_{i=1}^{n}\int_{-1}^{1} I_{ik} f_i(\xi)\vec{e}_{s_k'} \times \nabla g_{0k}(\vec{r},\vec{r}')\frac{ds_k'}{d\xi}d\xi \quad (17)$$

$$\vec{H}_{Ik}^e = -\frac{1}{4\pi}\sum_{i=1}^{n}\int_{-1}^{1} I_{ik}^e f_i(\xi)\vec{e}_{ks*} \times \nabla g_{ik}(\vec{r},\vec{r}*)\frac{ds_k'}{d\xi}d\xi \quad (18)$$

The reduction to the case of a single wire is straightforward, and can be found elsewhere, e.g. in [1].

IV. COMPUTATIONAL EXAMPLES

First example deals with related to the simple PLC circuit shown in Fig. 2. The distance between poles is L=200m, with the wire radius a=6.35mm. The wires are suspended on the poles at heights h_1=10m, and h_2=11m. The maximum sag of the conductor is s=2m. Ground parameters are ε_r=13 and σ=0.005S/m. The power of the applied voltage generator is 2.5μW and operating frequency is 14MHz. The terminating load impedance Z_L is 500Ω.

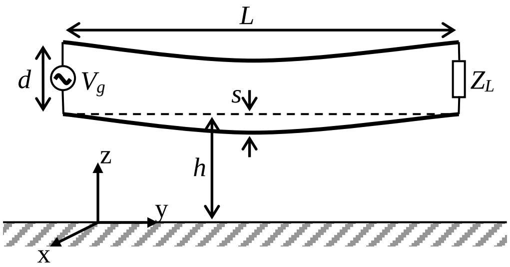

Fig.2. Simple PLC circuit

Radiated electric field at the distance of 30m from the wires and 10m above the lossy ground ground is shown in Fig 3.

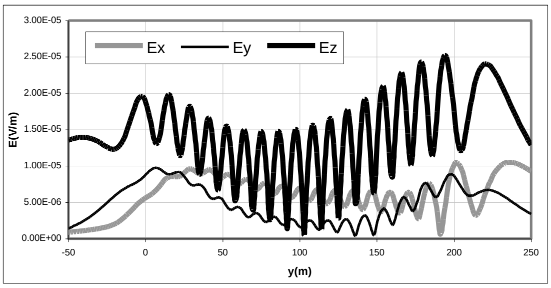

Fig. 3. Radiated electric field

Second example is related to simple PLC circuit shown in Fig. 4. The distance between the voltage source and terminating impedance at the other end is L=3.5m, with the wire radius a=2.25mm. The distance between wire axes is determined by the isolation thickness and is equal to 6.5 mmm. At the connections at both ends of the cable, the distance between the wires is 5cm. The amplitude of the voltage generator is 5V and operating frequency is 83.2 kHz (frequency of the SFSK P2LPC Mark signal). The terminating impedance Z_L is 3.7Ω. Fig 5 shows the distribution of the total near electric field in the plane 3m away from the wires.

It is worth noting that the electric field calculated 3m from the wires corresponds to power meters located on different floors in the building.

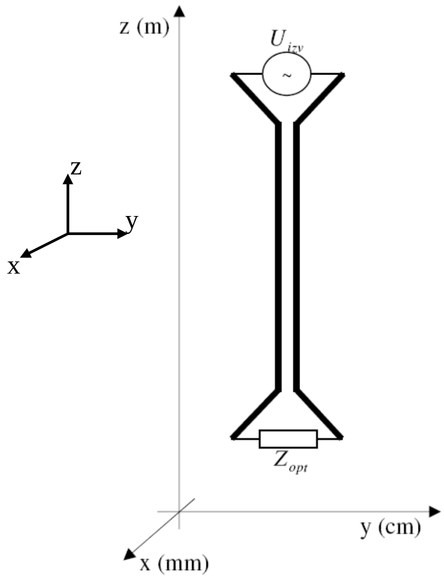

Fig. 4. Simple narowband PLC circuit

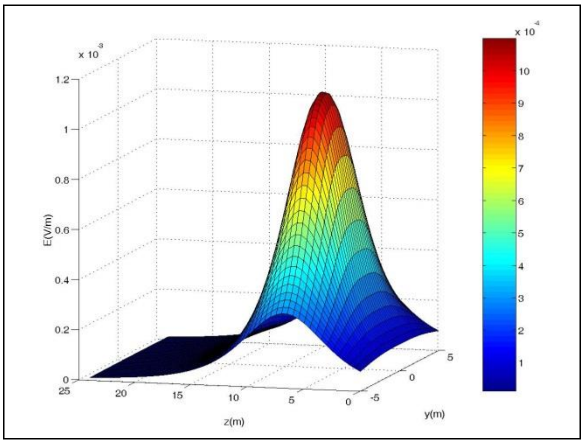

Fig. 5. Total E field distribution (x=3m, f=83.2kHz, Z_L=3.7Ω)

Analyzing the calculated results one figures out that the field levels are very close to the limitations proposed by the ITU-T K.60 standard. Compared to the German NB 30 or the UK standard, the obtained field levels exceed the limitations by 9dB and 24 dB, respectively. More details in this regard can be found in [5]. On the other hand the maximum levels of the electric field are 50 dB below the limits proposed by the American FCC Part 15 standard [10].

V. CONLUSION

The paper aims to analyze the electromagnetic field coupling to multiple wire configuration of arbitrary shape. The formulation is based on the antenna theory and the corresponding set of Pocklington integro-differential equations. The effects of the air-earth interface have been taken into account via the Fresnel plane wave reflection coefficient for TM and TE polarization, respectively. The solution of the Pocklington equations set is carried out via the Galerkin-Bubnov scheme of the Indirect Boundary Element Method (GB-IBEM). Some illustrative computational examples pertaining to PLC systems are presented in the paper.

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