Parameter estimation for low-order models of complex buildings

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Abstract—The improvement of the building sector energy efficiency becomes crucially important to attain a balance in many sectors. Reduction of the energy consumption in buildings by using model predictive control strategies is recognized as one of the essential solutions to achieve considerable energy savings. Due to the nature of thermodynamic processes in buildings the underlying models are mostly nonlinear and of high order. In this work Constrained Unscented Kalman Filter is employed to obtain a linear low order model of a large public building applicable for the predictive control. Through the comparison of results with the data generated by highly accurate building simulation software IDA Indoor Climate and Energy (IDA-ICE), it has been shown that the first order linear model for each zone, with separated nonlinearities related to the solar radiation effects, is sufficient to capture the main dynamics of the observed building.

Keywords—Constrained Unscented Kalman Filter (UKF), Building temperature prediction, Model Predictive Control (MPC), Online parameter identification, Parameter-Adaptive building model

I. INTRODUCTION

In the last few decades the global warming has become an important problem that influences the world energy market. Aware of this, the scientific, economic and politic communities started to advocate and encourage rational consumption of the energy resources. The building sector is crucial to the European Union (EU) environment and energy policies as buildings consume 40% of the total EU energy and generate about the same percentage of greenhouse gasses [1]. The heating, ventilation and air conditioning (HVAC) systems are responsible for majority of the overall energy consumed in buildings [1]. Use of advanced smart control methods to operate the HVAC systems is therefore crucially important to improve the energy efficiency in the building sector.

Model Predictive Control (MPC) framework has attracted considerable attention as a promising algorithm for smart energy efficient buildings. Its clear outstanding advantages make it suitable for the Building Energy Management System (BEMS) design. Traditional control algorithms mostly rely on the calibration of algorithms designed for a typical building according to approximate rules of thumb or trial and error methods. While this is sufficient to satisfy the environmental condition constraints set by the end users, it is not optimal in a sense of users comfort and power consumption. MPC is an advanced control strategy that relies on dynamic model of the process. In this way the control algorithm is strictly designed for the particular building. Potential building energy savings compared to the traditional rule based HVAC controllers are in the range from 16% to 41% [2]–[4]. Mathematical model of the building is a basis for the MPC implementation. Numerous software packages specialized for modeling of the building thermal behavior exist on the market. Although those programs can provide very accurate building models, the underlying models are mostly highly nonlinear and of a high order. The most popular building modeling framework consists of using the resistor-capacitor (RC) network to model thermodynamic processes in buildings. The RC network models are established as simple, computationally efficient and enough accurate models. The problem of RC representation is a fast increase in the model complexity with an increase of building zones. For large buildings the number of the system states can be over a couple of thousands. To be applicable for the control system design generally, the model of the process should be simple and yet accurate enough. While nonlinear and higher order models provide better accuracy, they are computationally too intensive for the control system design. Therefore, from the control viewpoint, the goal is to get a linear model of the lowest possible order in a way that the model uncertainty is lowest possible.

Besides physical modeling, building model can be determined by the use of advanced estimation techniques. Unscented Kalman Filter (UKF) is an algorithm that uses a series of noisy and inaccurate measurements observed over time to estimate unknown system states, parameters or even both. Use of UKF for estimation of building parameters was already reported in [5], [6]. Just like in the RC approach, the number of building parameters to estimate can be extremely high, which can further lead to inaccurate estimation of parameters. In this paper the strengths of both mentioned approaches are consolidated into a single algorithm in order to get the model of a large public building suitable for the control system design.

This paper is structured as follows: Section II in short describes the basic principles of the UKF and reasons for its use. In Section III the RC modeling framework is described and, based on it, the modeling structure is proposed. Results of the offline and online identification are given in Section IV. Finally, Section V concludes the paper.
II. UNSCENTED KALMAN FILTER

Kalman filter is an algorithm that estimates values of unknown states and/or parameters of a system from a series of noisy and inaccurate measurements taken over time on it. Early development of the Kalman filter dates back to the early sixties [7]. Since then the Kalman filter found its place in many applications; guidance, robotics, control, signal processing, etc. Over time many extensions and generalizations of the Kalman filter have been developed in order to adjust the filter to the specific types of systems. The most known extensions of the Kalman filter which can be applied to nonlinear systems are Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). Although both mentioned filters are suitable for nonlinear systems, it has been shown that UKF in most applications outperforms EKF [5], [8]. Instead of the analytical linearization of nonlinear functions, UKF uses a deterministic sampling approach to capture mean and covariance of the estimates with a minimal set of sample points. Since the spread of the variables is also taken into account, UKF can be accurate up to a second order in estimating the mean and covariance.

The following discrete-time nonlinear system is considered:

\[ x_k = f(x_{k-1}, u_k, u_{k-1}) + v_{k-1}, \]
\[ y_k = h(x_k) + w_k, \]

where \( x \) denotes the system state, \( v \) is the process noise, \( w \) is observation noise, \( u \) is the deterministic system input, \( y \) is the noisy system observation and \( f(\cdot) \) and \( h(\cdot) \) are nonlinear functions. For the identification of a continuous time system \( f(\cdot) \) represents integration of the continuous time function over a unit-sample time interval. For the case of simultaneous estimation of parameters and states, unknown parameters are treated as additional states. Even for the linear model case, simultaneous states and parameters estimation makes the resulting identification nonlinear, i.e. performed on a nonlinear state-update model. Detailed description of the UKF algorithm can be found in [9].

For the models based on physical laws, due to the large number of estimation parameters and/or unobservable model form, the model (1) may result in nonnegative parameters to be negative or vice versa, whilst this is not physical. UKF can overcome this problem by introducing the constraints [8], [10]. The candidates to be constrained are a-priori and a-posteriori state estimates, sigma points, sigma points propagated through the nonlinear function, etc. [10]. The main idea of the constrained UKF is to select the point \( \tilde{z} \) from the set defined by constraints that is closest to \( z \) by solving the following convex problem:

\[ \min_{\tilde{z}} (\tilde{z} - z)^T W_k (\tilde{z} - z), \]
\[ \text{s.t. } D_k \tilde{z} \leq d_k. \]

(2)

It can be shown that for \( W_k = I \) and \( D_k = I \), where \( I \) is the identity matrix, the solution of (2), in the case when \( z \) violates the constraints, is \( \tilde{z}_i = d_k,i \) for those constraints \( i \) that are violated [10]. More generally for constraints introduced as box constraints, solving (2) gives the same solution as clipping, i.e. setting all points outside the allowed set to the boundaries.

III. MATHEMATICAL MODEL

The model studied in this paper is a model of the 9th and 10th floor of the University of Zagreb Faculty of Electrical Engineering and Computing (FER) building. These two floors consist of 50 zones, mainly classrooms, laboratories and offices. Overall studied building area is about 3000 m². Figure 1 shows the 3D model of the 9th floor of the FER building. First, a detailed simulation model of the considered area is developed with highly accurate building simulation software IDA-ICE [11]. This high order nonlinear model is then used as a source of the identification data and benchmark model for the proposed modeling structure.

Fig. 1: 3D model of the 9th floor of FER building.

A. RC modeling framework

RC network models of a building are established as simple, computationally efficient and enough accurate models. The RC method began to develop in the early eighties [12]. The basic strategy of this methodology is to represent building elements (or complete zones) with as few thermal circuit elements as possible (lumping). Accuracy of the model depends on the number of capacitors used to describe the heat transmission through a wall. The analogy is as follows: heat flow is represented by current, temperatures are represented by voltages, thermal resistances are represented by resistors and thermal capacitances by capacitors.

Thermal resistance is defined as the temperature difference across the structure when a unit of heat energy flows through it in a time unit. Thermal resistance of convection between a fluid and a solid body is defined as:

\[ R_{conve} = \frac{1}{h \cdot A}. \]

(3)

where \( h \) is convective heat transfer coefficient and \( A \) is the area of the wall exposed to the heat transfer. Thermal resistance of conduction within a solid body is defined as:

\[ R_{cond} = \sum_{i=1}^{n} \frac{L_i}{k_i \cdot A}. \]

(4)

where \( n \) is the number of material layers that form a wall, \( L_i \) is thickness of the \( i \)-th layer and \( k_i \) is its thermal conductivity. In order to analyze dynamic behavior of the building it is necessary to introduce dynamic elements. Dynamic behavior of the building can be described with a heat capacity concept.
Heat capacity is the property of a body to store the heat energy.

In buildings with a large share of external windows it is necessary to pay a special attention to the modeling of solar radiation influence. Solar radiation can be absorbed by solid external surfaces or absorbed and transmitted into the zone through external windows. The rate of absorbed solar radiation depends on the material properties. Overall solar radiation transmitted into the zone through windows can be calculated from the known window property called Solar Heat Gain Coefficient (SHGC) (Hemispherically averaged SHGC for diffuse solar radiation) [13]. SHGC is the fraction of solar radiation admitted through a window - either transmitted directly and/or absorbed, and subsequently released as heat inside a zone. It is a nonlinear function of the solar incidence angle. There are two ways to obtain SHGC and its dependence on the solar inclination angle for the specified type of window. Tabular dependence of SHGC on the solar incidence angle for most common types of window systems can be found in ASHRAE handbook [13]. Second way is to obtain SHGC with free software tool, WINDOW from the Lawrence Berkeley National Laboratory [14]. Although the solar radiation model is strictly nonlinear [15], it belongs to a special class of nonlinearities called separable nonlinearities. This property allows extraction of the nonlinearity outside a model and use of its outputs as inputs into the linear model as shown in Figure 2.

RC model of the FER building, with external walls represented with 2 capacitors and zones and internal walls represented with one capacitor, has order over 500. Often it is very difficult to determine the thermal properties of walls and the exact values of convection coefficients required to accurately determine the parameters of the RC network. In such cases the application of identification techniques becomes necessary.

### B. Proposed modeling structure

The objective of this paper is to present the modeling structure which consolidates the strengths of both the RC approach and identification procedures and as a result gives the model applicable for the control system design. In order to fulfill this objective, the following state space model structure is proposed:

$$
\dot{x}(t) = A_t x(t) + B_t u(t),
$$

where $n$ is the number of building zones, $m$ is the number of the system inputs (outdoor temperature, pre-calculated absorbed and transmitted solar radiation, boundary conditions, etc.). Just like in the RC approach, without a-priori knowledge on the model structure, the number of parameters to estimate can be extremely high. Large number of parameters can result in computational inefficiency of the algorithm and inaccurate estimation of parameters. The number of estimation parameters can be reduced if the model structure to be identified is known (known positions of nonzero elements in system matrices). In the proposed structure (5) it is assumed that the dynamics of each zone can be approximated with first order differential equation:

$$
\frac{dT^j}{dt} = \alpha^j T^j + \sum_{i \in N^j} \beta^{ij} T^i + \gamma^j T_{\text{out}} + \sigma^j_1 I_{\text{a}}^j + \sigma^j_2 I_{\text{r}}^j,
$$

where $T^j$ is the temperature of the $j$th zone, $T_{\text{out}}$ is the outdoor air temperature, $I_{\text{a}}^j$ is the overall solar radiation to the external walls belonging to the $j$th zone and $I_{\text{r}}^j$ is the overall solar radiation transmitted into the zone through the windows. The set $N^j$ consists of the indices of zones adjacent to the $j$th zone. Greek letters $\alpha$, $\beta$, $\gamma$ and $\sigma$ denote the estimation parameters. For zones without external walls or without the external windows the corresponding terms in (6) are omitted.

Known model structure can significantly reduce the number of identification parameters, but for large buildings the number of parameters can still be too large for accurate identification. The main idea of this paper is to take advantage of the known physical parameters of the building such as walls materials and thicknesses. Construction properties of external walls are much more complicated than the construction properties of the internal walls. Internal walls of FER building are mostly composed from standard full brick and cement plaster. Thermal properties of those materials are well documented. For well-known thermal properties of internal walls the number of independent estimation parameters can be further reduced by introducing the basic principles of the RC framework into the proposed modeling structure. In a steady state the distribution of temperature through a solid wall is linear, i.e. the heat transfer between two adjacent zones is proportional to the temperature difference between those zones, where factor of proportionality is the overall heat resistance of a wall that separates them, $R_{ij}$. Thus for known

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**Fig. 2: Structure of the building model.**
parameters of internal walls, structure (6) can be rewritten as:

\[
\frac{dT_j}{dt} = \alpha_j T_j + \beta_j \sum_{i \in N_j} \frac{T_i}{R_{ij}} + \gamma_j T_{out} + \sigma_1^j I_j + \sigma_2^j I_j. \tag{7}
\]

The overall resistance related to the particular internal wall is defined as a sum of convective heat resistances between the outer and inner face of a wall and air in the corresponding zone and thermal resistance of conduction through the wall. If thermal resistance of some internal wall is initially wrongly determined, structure (7) assumes that the error acts proportionally to the accompanying thermal resistances on all internal walls related to the same zone. Model structure (7), with \( R_{ij} \) calculated using equations (3) and (4), for FER building has twice less parameters than structure (6).

IV. RESULTS

Two approaches to the identification of building parameters are tested: online and offline. Quality of the identification largely depends on quality of the data used for identification. The timeframe of the identification data which ensures the excitation of all system modes depends on climate of the location at which the building is placed. For the identification of FER building, placed in Croatia, the best identification data are those recorded during the spring or fall. Detailed building simulation model developed in IDA-ICE simulation software represents a benchmark model for the identified models. Also its zone temperature logs will be used as measurements required for identification. Data used for external conditions (outdoor temperature, global and diffuse solar radiation, relative humidity, wind speed and direction) in IDA-ICE model and for the purpose of identification are historical data from the nearest meteorological station to the FER building. Sampling time of all data used for identification is one hour. Dynamics of the building excited only by external environmental conditions is very slow which justifies the selection of the sampling time. The main advantage of the continuous-time identification approach, applied in this work, is the independence of identified model parameters on the sampling time, so the identified model can be used with an arbitrary sampling time. To ensure the physicality and stability of the proposed model the following constraints are integrated into the UKF algorithm:

\[
\alpha_j < 0, \quad \beta_j, \gamma_j, \sigma_1^j, \sigma_2^j > 0, \quad \forall j = 1, 2, ..., n. \tag{8}
\]

The comparison of the time-responses of the identified models and a benchmark model is presented through the comparison of the time-response of a single zone. The selected zone is south oriented, with a large share of external windows area. Such characteristics make it more exposed to the external conditions (more pronounced dynamics).

![Fig. 3: The comparison of the time responses of the IDA-ICE model and model identified with UKF (one year prediction horizon).](image-url)
A. Offline identification

Data used for offline identification cover the period from the beginning of March to the end of May. Performance of both models, IDA-ICE model and the identified model, are tested on the same input data over a whole year. The results are shown in Figure 3. For large buildings dynamics of the zones can significantly differ. This difference in dynamics is especially evident between zones facing south and zones facing north. The Root Mean Squared Error (RMSE) between the identified model and IDA-ICE benchmark model calculated for whole building (all 50 zones) is shown in Figure 4. Please note that the best model performance is exhibited in the months following the end of the identification period, which clearly motivates the permanent, on-line model adaptations. Figure 5 shows the comparison of the 24 hour prediction of the identified model and IDA-ICE value of zone temperature (“actual” zone temperature) for two days at the end of March. The dots on each line represent (from the left to the right) prediction of the model from 1h to 24h in advance. As it can be seen in Figure 5, the prediction error is below 1°C.

B. Online identification

Online identification of the model parameters results in Parameter-Adaptive (PA) building model. This model is able to sufficiently capture all building operation modes and to adapt the model parameters according to the current operation mode or current external conditions. Generally, buildings dynamics is highly nonlinear and time-varying. For instance, convective heat transfer coefficients for external walls highly depend on the wind speed and direction, radiation heat flux between walls is nonlinear. This nonlinear dependence can be captured with the adaptive parameters of the proposed linear model with respect to the current building operating conditions. Figure 6 represent a schematic view of the PA model included into the BEMS. Figure 7 shows the comparison of the 24 hour prediction of the PA model and the actual zone temperature. At each identification step the new PA model is generated using the latest set of identified parameters and exploited to calculate the 24 hour prediction of zone temperatures.

Expectedly online application of the UKF gives better results. Model parameters are tuned in every time step so the
resulting PA building model captures the building dynamics more accurately than offline identified model. Despite the very low order of the proposed modeling structure, the RMSE of both tested approaches is below 1°C. Shown accuracy and low order make presented model structure applicable for the design of the MPC.

The future work will include an extension of the proposed structure with actuators and additional thermal loads, such as occupancy or equipment loads. By separation of the identification into several procedures, one during night, when zones are unoccupied and not influenced by solar radiation, equipment or actuators, and others during day, when zones are affected by the mentioned gains, the additional heat loads will be satisfactorily estimated [6]. The continuous-time identification approach allows easy change of the sampling time; one hour sampling time is too large given the dynamics of the zones heated/coolied with the actuators. Independency of the identified model parameters on the sampling time is of a high importance in the hierarchical model predictive control, where control is distributed between two or more control levels with different sampling times.

V. Conclusion

Advanced control algorithms such as MPC are appropriate for the significant energy reduction in buildings. To tailor the MPC towards the building, its mathematical model is required. Knowledge of the mathematical model of the building allows prediction of the future building behavior. Buildings are complex nonlinear dynamic systems with uncertain and time-varying dynamics, therefore the development of the accurate and reliable building model is a challenging task. While nonlinear high order models provide better accuracy, they are computationally too intensive for the application in control systems. Nonlinearities, such as the calculation of the overall solar radiation to the external walls or overall radiation transmitted into the zone through window, can be separated from the model, calculated independently from the model and forwarded to the linear building model as system inputs. The proposed linear first order zone model, with exploited knowledge from the RC approach to reduce the number of independent estimation parameters, proved to be sufficient to capture the main dynamics of the building. The continuous-time identification approach ensures the independence of the identified model parameters on the sampling time which ensures applicability of the obtained model with various sampling times. Constrained UKF identification algorithm, due to its scalability, easy constraint handling and accuracy can be considered as a promising algorithm for the real implementation of building energy management system.

ACKNOWLEDGMENT

This work has been financially supported by the European Union through project ENHEMS-Buildings – Enhancement of Research, Development and Technology Transfer Capacities in Energy Management Systems for Buildings, grant No. IPA2007/HR/16IPO/001-040510.

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