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Book of Abstracts

On Asymptotic Behavior of Solutions to Higher Order Nonlinear Differential Equations

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Consider the equation $y^{(n)} + p(x, y, y', ... y^{(n-1)})\ |y|^k \mathrm{sgn} y = 0$ with $n \geq 1$, real $k > 0,\ k \neq 1$ and a continuous function p.
Some new results about the asymptotic behavior of blow-up and oscillatory solutions for $k > 1$ are obtained. For example, the existence of such solutions with non-power quasi-periodic behavior is proved for constant p. This yields the existence of solutions with arbitrary number of zeros. Similar results are also obtained for $0 < k < 1$.

References and Literature for Further Reading

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Stability of Cooperative Dynamics on Graphs under Time Delays

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We consider dynamics on finite graphs subject to a discrete Laplacian operator and time delays. We study the stability of the spatially homogeneous (synchronized) state and its relation to the properties of the underlying graph. We address both discrete and continuous-time systems, as well as discrete and distributed delays. Applications are indicated in the fields of the stability of traffic flow, synchronization of coupled oscillators, and consensus in social dynamics and distributed computing.

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On Limit Cycles in Liénard Type Equations

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We consider the Liénard type equations

$$x'' + f(x)x' + g(x) = 0$$

where $f(x),\ g(x)$ are polynomials, with respect to the existence and location of limit cycles.

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Darboux Problem on Multidimentional Time Scale

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We consider existence and uniqueness of solutions for Darboux problem of neutral type on cartesian product of time scales. Lipschitz conditions with respect to functional arguments of the right-hand side of equation are assumed. We use some Bielecki norm in the space of rd-continuous functions and Banach contraction principle.

A Differential Equation Model of Optimization of Data Transfer Rate

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We consider a system of differential equations that has a delay depending on the solution. The time delay is defined by an ordinary differential equation with a non-continuous right hand side. The problem emerges in optimization (in the function of utility and price) of data transfer rate of computer networks.
The equation can not be inserted neither in the standard theory of functional differential equations nor in the theory of equations with state-dependent delay emerging in the recent years. Two main technical problems cause the difficulty: the state-dependent delay and the not smooth member in the algebraic equation defining the delay.
The main result is that the system defines a continuous semi-dynamical system. Under certain conditions we verify global convergence to the optimum (which is an equilibrium). We also prove results for existence of periodic solutions around the optimum.
New results: construction of appropriate phase space for the problem, verifying existence and uniqueness of solution in the phase space and continuous dependence on initial data, furthermore showing possibility of periodic behavior.

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Permanence in the Nonautonomous Competitive Reaction–Diffusion System with Delays

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In this talk we consider a nonautonomous competitive reaction–diffusion system with delays

$$\begin{cases} \dfrac{\partial u_i}{\partial t} = \Delta u_i + f_i(t,x,u_t)u_i, & t > t_0,\ x \in \Omega,\ i = 1,\ldots,N \\ \alpha_i(x)u_i(t,x) + k_i(x)\frac{\partial u_i}{\partial n}(t,x) = 0, & t > t_0,\ x \in \partial\Omega,\ i = 1,\ldots,N, \\ u_i(\Theta,x) = \phi_i(\Theta,x) & t_0 - \tau \le \Theta \le t_0\ x \in \Omega, i = 1,\ldots,N \end{cases} \tag{RD}$$

where $u_t(\cdot,x)$ denotes the member of $C([-\tau,0]^N)$ defined by $\Theta \mapsto u(t+\Theta,x) = (u_1(t+\Theta,x),\ldots u_N(t+\Theta,x))$, $\alpha_i : \bar{\Omega} \to [0,\infty)$ and $k_i : \bar{\Omega} \to [0,\infty)$ are C^1, Ω is a bounded domain with a sufficiently smooth boundary $\partial\Omega$, Δ is the Laplace operator on Ω . Applying the Ahmad and Lazer's definitions of lower and upper averages of a function we give an average conditions for the permanence of the system. This results can be used to the ordinary differential equations.

Scale-Invariant Selfadjoint Extensions of Scale-Invariant Symmetric Operators: Differential Operator Versus Difference Operator

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We consider symmetric operators with index of defect $(1, 1)$ which are unirarily equivalent to their scalar multiple. Such operators we call scale-invariant. Examples of scale-invariant operators are provided in classes of differential and difference operators. A difference operator under consideration are generated by bilateral Jacobi matrix and it acts in the space $l^2(q; \mathbb{Z})$ of all sequences $\{x_n\}_{-\infty}^{\infty}$ with complex entries such that $\sum_{-\infty}^{\infty} q^n |x_n|^2 < \infty$, ($q > 1$ is a fixed number). We investigate existence of selfadjoint extensions of such operators and compare corresponding results for differential and difference operators.
Some of the results of the presentation were obtained in collaboration with Martin Bohner (USA), Mark Nudel'man (Ukraine), and Hristo Voulov (USA).

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New Global Exponential Stability Criteria for Nonlinear Delay Differential Systems with Applications to BAM Neural Networks

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We consider a nonlinear non-autonomous system with time-varying delays

$$\dot{x}_i(t) = -a_i(t)x_i(h_i(t)) + \sum_{j=1}^{m} F_{ij}(t, x_j(g_{ij}(t))), \quad i = 1, \ldots, m$$

which has a large number of applications in the theory of artificial neural networks. Via the M-matrix method, easily verifiable sufficient stability conditions for the nonlinear system and its linear version are obtained. Application of the main theorem requires just to check whether a matrix, which is explicitly constructed using the system's parameters, is an M-matrix. Comparison with the tests obtained by K. Gopalsamy (2007) and B. Liu (2013) for BAM neural networks illustrates novelty of the stability theorems.

Nonoscillation and Oscillation Properties for Third Order Difference Equations of Neutral Type

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We consider a class of third order nonlinear delay diffrence equations of neutral type. Some criteria for the oscillation of bounded solutions will be presented. An example illustrates the result.

References and Literature for Further Reading

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Non-autonomous Systems with Impulses and the Navier-Stokes Equation

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In this work, we investigate the existence and uniqueness of mild solutions for the two-dimensional Navier-Stokes equation with impulses. We also present some results of the theory of attractors for the Navier-Stokes equation with impulses.

References and Literature for Further Reading

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Cluster Formation in Granular Dynamics, Bird Flocking and Chimera States

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One of the most interesting applications of differential and difference equations concerns the mathematical modeling of complex systems arising in Physics and Biology. In this lecture, we will use differential and difference equation models to study the phenomenon of cluster formation, according to which such systems "condense", form flocks or synchronized patterns, apparently spontaneously during their time evolution. This type of behavior typically occurs as a result of an internal instability, due to which the system undergoes a global "bifurcation", that one may well characterize as a dynamical phase transition. In particular, we will first describe clustering in granular material flowing down an array of periodically shaken boxes [1] and then discuss groups of birds, where a "noise" parameter causes flocking to break down by a phase transition, whose order remains to date a hotly debated issue [2,3]. Finally, referring to the observation that in the brain of some mammals the sudden appearance of neighboring synchronous and asynchronous neuronal ensembles forms the so called "chimera state" [4,5], we will present results on *Hindmarsh Rose* models of neuron oscillators as well mechanical networks of coupled pendulum-like systems [6,7], where such fascinating chimera states can be analyzed in detail in terms of networks nonlinear differential equations.

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Hyers-Ulam Stability of Linear Equations of Higher Orders

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We present some basic motivations, definitions and results connected with the notion of Hyers-Ulam stability. Next, we show general methods for investigations of that stability of the linear (difference, differential, functional and integral) equations of higher orders. They can be expressed in the terms of fixed points in suitable function spaces. In numerous situations, the Hyers-Ulam stability of a particular case of such equations is a consequence of similar properties of the corresponding first order equations. We also provide some examples of simple applications of those methods.

A Spectral Criterion for Hyers-Ulam Stability of Periodic Difference Equations of Order Two

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Let $\mathbb{Z}_+$ be the set of all nonnegative integer numbers and let $(a_n), (b_n)$ be complex scalar valued 2-periodic sequences. Let consider the non-autonomous recurrence of order two

$$x_{n+2} = a_n x_{n+1} + b_n x_n, \quad n \in \mathbb{Z}_+, \qquad (a_n, b_n)$$

and the matrices

$$A_n := \begin{pmatrix} 1 & 1 \\ a_n + b_n - 1 & a_n - 1 \end{pmatrix}, \quad n \in \mathbb{Z}_+.$$

We prove that the recurrence (a_n, b_n) is Hyers-Ulam stable if and only if the monodromy matrix $T_2 := A_1 A_0$ has no eigenvalues on the complex unit circle.

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Melnikov Theory for Discontinuous Systems

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We present some recent development on Melnikov theory for non-smooth ODEs. We consider a system having a critical point O lying on a discontinuity surface $\mathcal{S}$, and a trajectory homoclinic to O. We assume that the system is subject to a non-autonomous perturbation and we look for conditions which are sufficient for the persistence of the homoclinic trajectory and for the existence of a chaotic pattern. An important issue will be to control the position of trajectories close to O and $\mathcal{S}$.
Non-smooth and discontinuous ODEs arise in many physical models, such as mechanical systems with dry friction or with impacts, and have received a great interest for their relevance in applications.
This is a joint work with J. Diblík and M. Pospíšil (Brno University of Technology) and M. Franca (Ancona).

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Chemostats with Random Inputs

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Chemostat refers to a laboratory device used for growing microorganisms in a cultured environment, and has been regarded as an idealization of nature to study competition modeling of mathematical biology. The simple form of chemostat model assumes that the availability of nutrient and its supply rate are both fixed. However, these assumptions largely limit the applicability of chemostat models to realistic competition systems. In this work, we relax these assumptions and study the chemostat models with random nutrient supplying rate or random input nutrient concentration. This leads the models to random dynamical systems and requires the concept of random attractors developed in the theory of random dynamical systems. We will report on the existence of uniformly bounded non-negative solutions, existence of random attractors and geometric details of random attractors for different value of parameters.

(The content of this talk is mainly based in a joint work with Xiaoying Han and Peter Kloeden)

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Dispersal, Disease and Evolution: Challenges and Opportunities in Computational, Mathematical and Theoretical Epidemiology

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Challenges and opportunities in the study of disease dynamics, control and evolution will be highlighted. Initially, I will trace the history of the field of mathematical epidemiology and the role of trade and mobility on the spread, transmission and evolution of infectious diseases such as influenza and tuberculosis. Emphasis will be placed on the role of multiple time scales and the study of long-term versus short-term dynamics. The role of dispersal and residence times will be briefly addressed.

On One Method of Investigation Linear Functional Differential Equations

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We present the results of study some scalar linear functional-differential equations of different types. Main attention is paid to the initial problem with the initial point, when the initial condition is specified at the initial point and the classical solution, whose substitution into the original equation transforms it into the identity, is searched for. The method of polynomial quasisolutions that is based on representation of the unknown function $x(t)$ as a polynomial of degree N is applied as the method for study. Substitution of this function in the original equation results in residual $\Delta(t) = O(t^N)$, for which a faithful analytical representation is obtained. In this case the polynomial quasisolution is understood as the exact solution in the form of the polynomial of degree N, disturbed because of the residual of the original initial problem. The results of the numerical experiments are presented.

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Second Order Semilinear Impulsive Differential Equations with Nonlocal Conditions

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An abstract second order semilinear differential equation such that the linear part of the right-hand side is given by the infinitesimal generator of a strongly continuous cosine family of bounded linear operators [1–3], and provided with impulse and nonlocal conditions is studied. Theorems for existence and uniqueness of a mild and classical solution of the problem considered generalizing the results of [2] are proved.

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Piecewise Linear Maps with Hysteresis

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We consider a simple one-parameter family of hybrid systems that consist of two affine oscillators. The switching strategy is hysteretic, which means that the partition of the phase space depends on which oscillator governs the trajectory. Then we construct a Poincaré-map, which preserves the hysteretic nature of the dynamics. This map is a piecewise linear map with overlapping domains, that is a multi-valued map. We study this map by invariant measures and symbolic dynamics.

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Stability and Instability on Elementary Way

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Mark Levi and Waren Weckesser gave a simple geometrical explanation for the stabilization of the upper equilibrium of the mathematical pendulum when its suspension point is vibrating vertically with so hight frequency that the gravity can be neglected. With my supervisor, we extended this method to a more natural case, when the effect of gravity is taken consideration, using simple geometric calculations.

Floquet Theory is a well-known idea which usefull to describe linear systems with periodic coeffitiens, e.g., the swing. I show how we can draw stability map on the parameter plane without Floquet theorem, using elementary geometric ideas.

References and Literature for Further Reading

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Point Symmetry and Differential Invariants for the Navier–Stokes Equations

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We consider the system of Navier–Stokes equations and its point Lie symmetry. We find the Lie algebra of symmetry of this system and using the Tresse theorem we construct a basis of differential invariants of the found Lie algebra. Further we discuss some invariant form of Navier–Stokes equations.

An Application of Scales of Banach Spaces and Theory of Interpolation

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Scales of Banach Spaces and interpolation spaces play an important role in modern partial differential equations. The aim of this presentation is to show the construction of scale of Banach space and different methods to construct interpolation space. Furthermore, their properties will be demonstrated, together with the theorem containing conditions which guarantee that spaces on the scale are equal to the interpolation spaces. In the end, the applications to solve some partial differential equations will be presented.

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Examples of Solvable q-Schrödinger Equations

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By solving an infinite nonlinear system of q-difference equations one constructs a chain of q-difference operators. The eigenproblems for the chain are solved and some applications, including the one related to q-Hahn orthogonal polynomials, are discussed. It is shown that in the limit $q \to 1$ the present method corresponds to the one developed by Infeld and Hull.

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On the Bounded and Stabilizing Solution of a Generalized Game Theoretic Riccati Differential Equation with Periodic Coefficients of Stochastic Control

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We consider a system of coupled matrix nonlinear differential equations arising in connection with the solution of an H_∞ type control problem for a system modeled by an Ito differential equation subject to random switching according with a standard homogeneous Markov process with a finite number of state. The system of differential equations under consideration contains as special cases the game theoretic Riccati differential equations arising in the solution of the H_∞ control problem from the deterministic case.

Among the global solution of the generalized game theoretic Riccati equation, an important role in the construction of the solution of the H_∞ control problem is played by the so called *stabilizing solution*.

In this work we present a set of conditions which guarantee the existence and uniqueness of the bounded and stabilizing solution of the Riccati differential equation under consideration. Also, we shall provide a method for numerical computation of this bounded and stabilizing solution. It is worth mentioning that we do not know a priori neither an initial value nor a boundary value of the bounded and stabilizing solution of the Riccati differential equation under investigation. That is why, the numerical methods applicable for the approximation of the solution of a Cauchy problem or of a boundary value problem associated to a differential equation cannot be used to compute the bounded and stabilizing solution of a Riccati differential equation. The bounded and stabilizing solution of the generalized game theoretic Riccati differential equation is obtained as a limit of a sequence of bounded and stabilizing solutions of some Riccati differential equations with defined sign of the quadratic parts. For this kind of Riccati differential equation exit reliable iterative procedures to obtain the stabilizing solution.

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Direct and Inverse Problems of the Calculus of Variations on Time Scales

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We consider some problems of the calculus of variations on time scales. First of all, we prove Euler–Lagrange type equations and transversality conditions for generalized infinite horizon problems. Next we consider a composition of a certain scalar function with the delta and nabla integrals of a vector valued field. Attention is paid on an inverse extremal problem for variational functionals on arbitrary time scales. We start by proving a necessary condition for a dynamic integro-differential equation to be an Euler–Lagrange equation. New and interesting results for the discrete and quantum calculus are obtained as particular cases. Furthermore, using the Euler–Lagrange equation and the strengthened Legendre condition, we derive a general form for a variational functional that attains a local minimum at a given point of the vector space. In the end, two main issues of application of time scales in economic with interesting results are presented. In the former case we consider a firm that wants to program its production and investment policies to reach a given production rate and to maximize its future market competitiveness. The latter problem relates to inflation and unemployment, which inflict a social loss. Using relations between p, u, and the expected rate of inflation π, we rewrite the social loss function as a function of π. Then, we apply the calculus of variations in order to find an optimal path π that minimizes total social loss over a given time interval.

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Extremal Estimates for the First Eigenvalue of the Sturm - Liouville Problem with Integral Condition

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Consider the Sturm-Liouville problem

$$y''(x) + Q(x)y(x) + \lambda y(x) = 0, \quad y(0) = y(1) = 0,$$

with a nonnegative bounded on $[0, 1]$ function $Q(x)$ satisfying the condition

$$\int_0^1 Q^\alpha(x)dx = 1, \quad \alpha \neq 0.$$

We estimate the first eigenvalue λ_1 of this problem for different values of α.
The variational principle implies that the first eigenvalue λ_1 can be found as

$$\lambda_1 = \inf_{y(x)\in H_0^1(0,1)} \frac{\int_0^1 y'^2(x)dx - \int_0^1 Q(x)y^2(x)dx}{\int_0^1 y^2(x)dx}.$$

Upper and lower estimates for λ_1 are obtained for different values of α.

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Spline Method for Singular Integro-Differential Equations

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For the periodic singular integro-differential equations

$$x^{(m)}(t) + \sum_{\nu=0}^{m-1}(a_\nu(t)x^{(\nu)}(t) + \frac{b_\nu(t)}{2\pi}\int_0^{2\pi} x^{(\nu)}(\tau)\cot\frac{\tau - t}{2}d\tau) + \gamma = y(t),\ m \geq 1,$$

subject to

$$\int_0^{2\pi} x(\tau)d\tau = 0,$$

we examine unique solvability conditions, justify a spline method for solving such equations, and present the numerical results obtained by solving a particular equation of this class.

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Oscillatory Solutions of Nonlinear Difference Equations Caused by Deviating Arguments

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In this paper we consider the oscillatory behavior of solutions of the nonlinear difference equation

$$(-1)^z \Delta^m y(n) = f(n, y(r_1(n)), y(r_2(n)), \ldots, y(r_k(n))),$$

where $z, \ k \in \mathbb{N}$ and $m \geq 2$ are given numbers, $n \in \mathbb{N}_{n_0}$, $f : \mathbb{N}_{n_0} \times \mathbb{R}^k \to \mathbb{R}$ and $r_i : \mathbb{N}_{n_0} \to \mathbb{N}_{n_0}$ are given functions such that $\lim_{n\to\infty} r_i(n) = \infty \ (i = 1, 2, \ldots, k)$.

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Hydrodynamic Flow Profiles
What Physicists Learn From Navier-Stokes' PDEs

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The Navier-Stokes equations, which describe Newtonian viscous fluid flow, are vector partial differential equations with a simple looking but highly nontrivial nonlinearity. This is of second order in the velocity field and in addition couples the local flow field gradient with the local flow direction. Except for some special cases of laminar flow the Navier-Stokes equations are tractable only by direct numerical simulations, since the flows of interest are strongly turbulent. – Most interesting features of turbulent fluid flow are their time averaged profiles. For these one can draw conclusions by time averaging the Navier-Stokes PDEs too. This leads to some *exact relations* useful for the proper interpretation of the measured profiles. Recent new insight for pipe flow, Taylor-Couette flow and Rayleigh-Bénard flow and close analogies between them will be reported in the talk. This in particular will shed new light on the so called "law of the wall", i. e., the Pr! andtl and von Kármán logarithmic boundary layer profile, and how one can go beyond by introducing a generalized turbulent viscosity to properly describe the Reynolds stress and calculating the profiles. The obtained relations may also serve to measure the properties of the generalized turbulent viscosity.

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Chaotic Iteration Based Pseudorandom Number Generators

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We will present a review of recent results obtained in the field of truly chaotic finite state machines and their applications to chaotic iterations based pseudorandom number generators, CIPRNG in short. Theoretical foundations taken from the study of iterative systems in the mathematical theory of chaos is firstly recalled in this state of the art. Practical aspects are then discussed to show the possibility and efficiency of the approach. All the CIPRNG proposed these last five years in the literature are then presented in detail and with references, and statistical results formerly published [1-4] are finally summarized.

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Spectral Theory for a System of Sturm–Liouville Differential Equations with Different Orders

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We develop the spectral theory for a system of self-adjoint Sturm–Liouville differential equations with different orders, in particular for a system with two equations of orders four and two. This corresponds to the fourth order self-adjoint Sturm–Liouville matrix differential equation, whose leading coefficient is singular. We prove that this system possesses all the traditional spectral properties of self-adjoint eigenvalue problems, such as the equality of the geometric and algebraic multiplicities of the eigenvalues, orthogonality of the eigenfunctions, the oscillation theorem and Rayleigh principle, and the Fourier expansion theorem. As a main tool we utilize the theory of finite eigenvalues and a new transformation of this equation into a linear Hamiltonian system.

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Suspension Bridges with Almost Distinct Ropes: Bifurcation Results

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We consider a modified version of a suspension bridge model originally introduced by Lazer and McKenna. In particular, we deal with the periodic PDE problem

$$\begin{cases} u_{tt} + u_{xxxx} + b\,r(x)u^+ = h(x) \quad \text{in } (0,1) \times \mathbb{R}, \\ u(0,t) = u(1,t) = u_{xx}(0,t) = u_{xx}(1,t) = 0, \\ u(x,t) = u(x,-t) = u(x,t+2\pi). \end{cases}$$

Here, the term $b\,r(x)u^+$ represents the nonlinear restoring force due to the suspension bridge cables with the stiffness b and density function r. The original model considered $r(x) \equiv 1$. Letting $0 \leq r(x) \leq 1$, we can model the "distinct distribution" of the cables. We study mainly the qualitative properties of the model and compare the cases of constant and non-constant density $r(x)$. In particular, we focus on the existence of multiple solutions. We show that for certain values of b, the bifurcation occurs. Moreover, we can expect also the blow-up effects, whose existence is closely connected with the so called Fučík spectrum of the corresponding linear differential operator.

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Generalization of Bistable Equation and Related Graph Theory Problem

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The bistable equation $u_t = \varepsilon^2 u_{xx} - F'(u)$ is perhaps the simplest model of phase transition at given critical temperature. The function F represents free energy and usually takes the form of a double-well potential. In [1], Drábek and Robinson offered an alternative explanation of *slow dynamics* by showing that continua of stationary solutions occur when the potential loses the C^2 continuity in its minimizers. Inspired by this we generalize the result for potentials of other types. A special choice of F then leads to a basic graph theory problem. On the way to its solution we reveal some interesting connections to other fields of mathematics.

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Determining the Dose of Antihypertensive Drugs by Differential Equations

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Captopril is an inhibitor, used for the treatment of hypertension and some types of congestive heart failure. This flow of medication operates by treating the parts of the body as compartments, so that a unit of the medication leaves one compartment and enters to another until eliminates from the body. This process is modeled by linear differential equations. The rates of absorption and elimination of drug is computed at each stage and is compared with together. Solving IVP's determines better dose for treating different diseases on variety conditions.

Applications of Difference Equations in Modelling Subdiffusion–Reaction Processes

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We use a random walk model in which both time and space variables are discrete in order to describe subdiffusion processes in which particles can chemically react. The new procedure which provides the Green's function for a subdiffusion–reaction process is proposed. The procedure can be briefly described as follows. We assume difference equations describing the random walk process in a system with both discrete time and space variables. We solve these equations by means of the generating function method. Next, we pass from discrete to continuous variables using the special formulae. We also show potential applications of the obtained functions to describe processes occurring in nature.

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Control in Systems of Linear Partial Differential Equations with Constant Delay

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We consider control problems for systems of linear partial differential equations with a constant delay. We describe a method of constructing the control function and to give formal solution of the control problem. We will use the special delayed functions in order to analytically solve auxiliary initial problems arising when the Fourier method is applied for ordinary linear differential equations of with a single delay. We find a control function added in the right-hand side of equations, for which at the finite moment of time the solution is equal to the given function.

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Variants of Average Shadowing

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The notion of shadowing has borne fruit to a wide range of variants and generalizations. Of particular interest are the concepts of shadowing in average, which are, unlike their predecessors, of global character. The talk will discuss two variants of average shadowing, their properties, and the differences between them. We will also present a unifying approach which enables us to prove certain properties sumultaneously for many types of shadowing.

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On Poisson Problem for a Linear Functional Differential Equation

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We study the Poisson problem

$$-\Delta u - \int\limits_{\Omega} (u(y) - u(x))\, r(x, dy) = f,$$
$$u\big|_{\Gamma(\Omega)} = 0,$$

where $\Delta u = u''_{x_1 x_1} + \cdots + u''_{x_n x_n}$, $x = (x_1, \ldots, x_n)$, Ω is an open set in $\mathbb{R}^n$, $\Gamma(\Omega)$ is the boundary of the Ω, under certain natural symmetry conditions. For almost all $x \in \Omega$, $r(x, \cdot)$ is an arbitrary measure. This problem has a clear mechanical interpretation. It has a unique solution for any $f \in L_2(\Omega)$. If $f \geq\not\equiv 0$, the solution $u > 0$ in $\Omega \setminus \Gamma(\Omega)$. The eigenvalue problem

$$-\Delta u - \int\limits_{\Omega} (u(y) - u(x))\, r(x, dy) = \lambda u,$$
$$u\big|_{\Gamma(\Omega)} = 0$$

has discrete spectrum of positive numbers, its eigenfunctions form an orthogonal basis in a Hilbert space $W \subset L_2(\Omega)$.

Causal Differential Equations on Time Scales

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The term of causal operators is adopted from engineering literature and the theory of these operators has the powerful quality of unifying ordinary differential equations, integro-differential equations, Volterra integral equations, and neutral functional equations, to name but a few. The study of functional equations with causal operators has seen a rapid development in the last few years and some results are assembled in a recent monograph [1]. In [2] Stefan Hilger has initiated the study of time scales which unifies the continuous and discrete cases. Since then many authors have studied qualitative properties of some nonlinear dynamic equations on time scales see [3]. In this paper, we estabilishe the existence of solutions and some properties of set solutions for the following Cauchy problem

$$\begin{cases} u^{\Delta}(t) = (Qu)(t), \ t \in [0,b)_{\mathbb{T}} \\ u(0) = u_0, \end{cases}$$

where $\mathbb{T}$ is a time scale (nonempty closed subset of real numbers $\mathbb{R}$), $[0,b)_{\mathbb{T}} = [0,b) \cap \mathbb{T}$, $b > 0$, and $Q : C([0,b)_{\mathbb{T}}, \mathbb{R}^n) \to L^p_{loc}([0,b)_{\mathbb{T}}, \mathbb{R}^n)$ is a causal operator.

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Entropic Inequalities for Finite and Infinite Matrices

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We present a review of our recent results expressed in a form of new matrix inequalities for Hermitian matrices. These inequalities are connected with known entropic inequalities for density matrices of composite quantum systems. The known inequality called the subadditivity condition for von Neumann entropy of the state of bipartite quantum system, for example, for two qubits, and the entropies of its subsystems provide the possibility to generalize the matrix expression of the inequalities given in an explicit form and obtain new matrix inequalities for Hermitian matrices. Another known entropic inequality called the strong subadditivity condition for a state of the three-partite quantum system written in an explicit matrix form for the density matrix of the composite system can be also generalized and a new inequality for an arbitrary Hermitian matrix can be obtained. The main tool to obtain the matrix inequalities is to use invertible maps of the integers $0, 1, 2, \ldots$ onto the pairs of integers $(00), (01), (10), (11), \ldots$ or triples of integers $(000), (001), (010), (100), \ldots$ Using these maps, arbitrary matrix elements of the same matrix A can be written in different forms, either as A_{jk} or $A_{mn,m'n'}$ or $A_{mnl,m'n'l'}$, etc. These different forms provide the possibility to construct the linear maps of the matrix A on the matrix A', i.e., $A \to A'$ by using a partial tracing expressed as a sum of the introduced indices. For example, the matrix A_{jk} expressed as the matrix $A_{mn,m'n'}$ yields the map $A \to A'$, where $A'_{mm'} = \sum_n A_{mn,m'n}$. For qutrit density matrices ρ and σ, one has the inequality

$$\mathrm{Tr}\left[\rho \ln(\rho\sigma^{-1})\right] \geq$$
$$\mathrm{Tr}\left\{\begin{pmatrix} \rho_{11}+\rho_{33} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \ln\left[\begin{pmatrix} \rho_{11}+\rho_{33} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}\begin{pmatrix} \sigma_{11}+\sigma_{33} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}^{-1}\right]\right\}.$$

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Probability Representation of Density Operators in Hilbert Space and Quantum Evolution Equations

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A review of the probability representation of quantum and classical states is presented. The method of star-product quantization to map the operators in a Hilbert space, including the density operators of quantum states and the operators corresponding to physical observables, is considered. The examples of particular invertible maps, for which the state density operators become standard nonnegative measurable probability distributions, are given for both continuous variables like positions and momenta of stationary and nonstationary coupled oscillators and discrete spin variables (qudits). The role of integral Radon transform providing the possibility to associate both the quantum particle state and the classical particle state with measurable tomographic probability distribution called the optical tomogram is elucidated. The kinetic classical equation (Liouville equation) and quantum evolution equations (von Neumann equation and Wigner–Moyal equation) are written as kinetic equations for the optical tomogram. The known solutions of the Schrödinger equation for coupled parametric oscillators, based on the existence of dynamical linear integrals of motion for the system of oscillators are presented in the form of probability distributions determined by the dynamical invariants, including the solutions of the Gaussian form (normal distributions). New entropic and information inequalities for the spin-state tomographic-probability distributions are obtained. New uncertainty relations for the tomographic distributions, which can be checked experimentally, and some of them, which were recently experimentally checked, are discussed.

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Tangential Center Problem and Pseudo-Abelian Integrals

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We consider planar polynomial différential equations having a center (i.e. singularities surrounded by a continuous family of periodic solution) and their deformations. We assume that the center is of Darboux type i.e. has a first integral of the form $F(x,y) = \Pi_{i=0}^{k} f_i(x,y)^{\lambda_i}$, with f_i polynomials and $\lambda_i > 0$.
Under generic assumptions, we give necessary and sufficient conditions, for the preservation of the center to first order in the deformation. The result is a joint work with Colin Christopher (Plymouth University, UK).

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Impulsive Neutral Functional Dynamic Equations on Time Scales

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This is a joint work with professors Márcia Federson, Miguel Frasson and Patrícia Tacuri. In this work, we study the relation between measure neutral functional differential equations, impulsive measure neutral functional differential equations, and impulsive neutral functional dynamic equations on time scales. For both types of impulsive equations, we obtain results on the existence and uniqueness of solutions and continuous dependence.

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Influence of Switching Curve Shape on Structure of Attraction Domain in Planar Filippov System

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We consider the one-parameter family of two-dimensional control systems with unlimited parametric perturbation. The problem is to find a limited discontinuous control law for this type of systems, allowing to stabilize each system of this family in a limited domain. It is assumed that any closed system of this family is a planar Filippov system [1] with unlimited right side.
We find a range of values of the parameter, for which there is a limit unstable cycle. This cycle defines the boundary of the origin attraction domain. Also we find a range of values of the parameter, for which the origin attraction domain loses its connectedness and the length of its boundary tends to infinity, while its area is limited. Sometimes, these regions are called fractal basin. However, in this case, there is no obvious self-similarity and therefore the use of this term in not correct.
Quasi-time optimal curve has infinite derivatives in 3 points. Two of points cause additional difficulties in the analysis of the phase portrait. To avoid these difficulties, we propose a curve that approximates it. This curve preserves the maximum topological proximity of the phase portrait for fixed values of the parameter. The latter means that the approximating curve must ensure the requirements. It provides finite-time stabilization of the origin, crosses axis Ox at the same points as the original curve, creates unstable limit cycle for the same values of the parameter as the original curve. To construct such curve we use polynomials with fractional powers. The numerical simulation results confirm the validity of this approximation.

This work was supported by Russian Foundation for Basic Research, grant No.13-01-00347 and Program no. 1 Scientific foundations of robotics and mechatronics

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The Fučík Spectrum for Differential and Difference Operators

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We investigate qualitative properties of the Fučík spectrum for differential operators of the second order and their discrete variants. We would like to point out the differences between continous and discrete cases. Results in [1] in an abstract general setting can be applied (inadmissible areas, tangent lines and asymptotes of the Fučík curves). Using the matching-extension method, we obtain the description of particular Fučík curves in the case of difference operators. Moreover, we discuss the solvability of the corresponding nonlinear boundary value problems at resonance with respect to the Fučík spectrum.

Presented results are available on a poster.

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Banach Lie-Poisson Spaces and Related Differential-Difference Equations

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Some hierarchies of Hamiltonian equations on the Banach Lie-Poisson spaces obtained by coinduction and induction procedures as well as by the Magri method will be presented. The operator equations of Riccati-type and semi-infinite Toda lattice are included in these hierarchies. One could consider the above hierarchies as systems of differential-difference equations. Applying the method of orthogonal polynomials we solve these equations for a few particular cases.

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Semigroups of Operators and Abstract Dynamic Equations on Time Scales

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We develop the theory of strongly continuous semigroups (C_0-semigroups) of bounded linear operators from a Banach space X into itself. Many properties of a C_0-semigroup $\{T(t) : t \in \mathbb{T}\}$ and its generator A are established. Here $\mathbb{T} \subseteq \mathbb{R}^{\geq 0}$ is a time scale endowed with an additive semigroup structure. We also establish necessary and sufficient conditions for the dynamic initial value problem

$$\begin{cases} x^{\Delta}(t) = Ax(t), & t \in \mathbb{T} \\ x(0) = x_0 \in D(A), \end{cases}$$

to have a unique solution, where $D(A)$ is the domain of A. Finally, we unify the continuous Hille-Yosida-Phillips Theorem and the discrete Gibson Theorem.

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On the Logistic Equation in the Complex Plane

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The famous logistic differential equation is studied in the complex plane. The method used is based on a functional analytic technique which provides a unique solution of the ODE under consideration in $H_2(\mathbb{D})$ or $H_1(\mathbb{D})$ and gives rise to an equivalent difference equation for which a unique solution is established in ℓ_2 or ℓ_1. For the derivation of the solution of the logistic differential equation this discrete equivalent equation is used. The obtained solution is analytic in $\{z \in \mathbb{C} : |z| < T\}$, $T > 0$. Numerical experiments were also performed using the classical 4th order Runge–Kutta method. The obtained results were compared for real solutions as well as for solutions of the form $y(t) = u(t) + iv(t)$, $t \in \mathbb{R}$. For $t \in \mathbb{C}$ the solution derived by the present method, seems to have singularities, i.e. points where it ceases to be analytic, in certain sectors of the complex plane. These sectors, depending on the values of the involved parameters, can move at different directions, join forming common sectors, or pass through each other and continue moving independently. Moreover, the real and imaginary part of the solution seem to exhibit oscillatory behavior near these sectors.
*Jointly with Efstratios E. Tzirtzilakis

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Resonance Phenomenon for a Certain Class of Second Order Differential Equations

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There will be presented asymptotic formulas for solutions of the equation

$$y'' + (1 + \varphi(x))y = 0, \;\; 0 < x_0 < x < \infty,$$

where function φ is small in a certain sense for large values of the argument. Usage of method of L-diagonal systems allows to obtain various forms of solutions depending on the properties of function φ.
The main aim will be discussion about the second order differential equations possesing a resonance effect known for Wigner-von Neumann potential. A class of potentials generalizing that of Wigner-von Neumann will be presented.

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Is Differential Transform Method Suitable to Solve Cauchy Problem for Delayed Differential Equations?

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Differential transformation method (DTM) has its origins in 1980's and it seems to be promising semi-analytical method of approximation of solutions of differential equations. The idea of the method is based on Taylor expansion.
Question of our interest is how to use DTM to solve Cauchy problem for delayed differential equations. Some attempts were made in this direction recently. We will introduce a novel approach of treating this problem and compare it to known DTM concept as well as to frequently used Adomian decomposition method.

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Asymptotic Equivalence of Impulsive Differential Equations

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Ws consider the following system of impulsive differential equations in Banach space $\mathbf{X} \times \mathbf{Y}$:

$$\begin{cases} dx/dt &= A(t)x + f(t,x,y), \\ dy/dt &= B(t)y + g(t,x,y), \\ \Delta x\big|_{t=\tau_i} &= x(\tau_i+0) - x(\tau_i-0) = C_i x(\tau_i - 0) + p_i(x(\tau_i-0), y(\tau_i-0)), \\ \Delta y\big|_{t=\tau_i} &= y(\tau_i+0) - y(\tau_i-0) = D_i y(\tau_i - 0) + q_i(x(\tau_i-0), y(\tau_i-0)), \end{cases} \tag{1}$$

satisfying the *conditions of separation*

$$\nu = \max\left(\sup_s \left(\int_{-\infty}^{s} |Y(s,t)||X(t,s)|\,dt + \sum_{\tau_i \le s} |Y(s,\tau_i)||X(\tau_i - 0, s)|\right),\right.$$

$$\left.\sup_s \left(\int_{s}^{+\infty} |X(s,t)||Y(t,s)|\,dt + \sum_{s < \tau_i} |X(s,\tau_i)||Y(\tau_i - 0, s)|\right)\right) < +\infty,$$

$f(t,\cdot), g(t,\cdot), p_i, q_i$ are ε-Lipshitz, $f(t,0,0) = p_i(0,0) = 0$, $g(t,0,0) = q_i(0,0) = 0$. Our goal is to find a simpler system of impulsive differential equations that is conjugated and asymptotic equivalent to the given one. Using this result we obtain sufficient conditions that noninvertible system (1) is asymptotic equivalent to the linear one

$$\begin{cases} dx/dt &= A(t)x, \\ dy/dt &= B(t)y, \\ \Delta x\big|_{t=\tau_i} &= C_i x(\tau_i - 0), \\ \Delta y\big|_{t=\tau_i} &= D_i y(\tau_i - 0), \end{cases} \tag{2}$$

in the case when ε depends on t and tends to zero as $t \to +\infty$ sufficiently rapidly.
This work was partially supported by the grant Nr. 345/2012 of the Latvian Council of Science.

Epsilon-Neighborhoods of Orbits of Parabolic Germs and Cohomological Equations

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We consider germs of parabolic diffeomorphisms $f : (\mathbb{C}, 0) \to (\mathbb{C}, 0)$ with the multiplier equal to 1. The question is if we can recognize a germ using the (directed) areas of epsilon-neighborhoods of discrete orbits. We have shown that the formal class can be read from finitely many terms in the asymptotic expansion in epsilon for any orbit. In this talk, we discuss analyticity properties of functions of (directed) areas of epsilon-neighborhoods of orbits. In particular, we concentrate on the coefficient of the quadratic term in the expansion, as a function of the initial point. It satisfies a cohomological difference equation similar to the trivialisation equation.

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Quantum Localization of Chaotic Eigenstates and the Statistics of Energy Spectra

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Quantum localization of classical chaotic eigenstatets is one of the most important phenomena in quantum chaos, or more generally - wave chaos, along with the characteristic behaviour of statistical properties of the energy spectra. Quantum localization sets in, if the Heisenberg time t_H of the given system is shorter than the classical transport times of the underlying classical system, i.e. when the classical transport is slower than the quantum time resolution of the evolution operator. The Heisenberg time t_H, as an important characterization of every quantum system, is namely equal to the ratio of the Planck constant $2\pi\hbar$ and the mean spacing between two nearest energy levels ΔE, $t_H = 2\pi\hbar/\Delta E$.

We shall show the functional dependence between the degree of localization and the spectral statistics in autonomous (time independent) systems, in analogy with the kicked rotator, which is the paradigm of the time periodic (Floquet) systems [7], and shall demonstrate the approach and the method in the case of a billiard family [8,9] in the dynamical regime between the integrability (circle) and full chaos (cardioid), where we shall extract the chaotic eigenstates. The degree of localization is determined by two localization measures, using the Poincaré Husimi functions (which are the Gaussian smoothed Wigner functions in the Poincaré Birkhoff phase space), which are positive definite and can be treated as quasi-probability densities. The first measure A is defined by means of the information entropy, whilst the second one, C, in terms of the correlations in the phase space of the Poincaré Husimi functions of the eigenstates. Surprisingly, and very satisfactory, the two measures are linearly related and thus equivalent.

One of the main manifestations of chaos in chaotic eigenstates in absence of the quantum localization is the energy level spacing distribution $P(S)$ (of nearest neighbours), which at small S is linear $P(S) \propto S$, and we speak of the linear level repulsion, while in the integrable systems we have the Poisson statistics (exponential function $P(S) = \exp(-S)$), where there is no level repulsion ($P(0) = 1 \neq 0$). In fully chaotic regime with quantum localization we observe that $P(S)$ at small S is a power law $P(S) \propto S^\beta$, with $0 < \beta < 1$. We shall show that there is a functional dependence between the localization measure A and the exponent β,

namely that β is a monotonic function of A: in the case of the strong localization are A and β small, while in the case of weak localization (almost extended chaotic states) A and β are close to 1.
We shall illustrate the approach in the model example of the above mentioned billiard family, where we can separate the regular and chaotic states. This presentation is based on our very recent papers [1,4,5,7].

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Oscillation Theorems for Second Order Nonlinear Neutral Delay Differential Equations

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We are concerned with the oscillatory behavior of a class of second-order nonlinear neutral delay differential equations

$$(r(t)\,|z'(t)|^{\alpha-1}\,z'(t))' + q(t)f(x(\sigma(t))) = 0, \tag{1}$$

where $t \geq t_0 > 0$, $z(t) := x(t)+p(t)x(\tau(t))$, and α is a positive constant. Oscillation of equation () has been studied recently by Ye and Xu [2]. In a special case where $f(u) := |u|^{\alpha-1}u$, equation () reduces to a quasilinear neutral differential equation

$$(r(t)\,|z'(t)|^{\alpha-1}\,z'(t))'x(\sigma(t)) = 0 \tag{2}$$

which was studied by Sun et al. [1] and Zhong et al. [3]. In this talk, we discuss improvements of several results obtained for equations (1) and (2) in the cited papers.

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On Symmetric Solutions of a Class of Weakly Nonlinear Systems

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A weakly non-linear ordinary differential system

$$x'(t) = \varepsilon f(x(t), t), \qquad t \in J, \tag{*}$$

is considered, where $J \subset \mathbb{R}$ is an open interval, $\varepsilon \in \mathbb{R}$ is a small parameter and the function $f : \mathbb{R}^n \times J \to \mathbb{R}^n$ is C^2-smooth and symmetric in a certain sense.
A related notion of symmetry of a solution of (*) is introduced. The existence and uniqueness of a symmetric solution is established and conditions for its stability are analysed. The property of the solution in question includes, in particular, the cases of periodic, antiperiodic, even, and odd solutions.
The talk is based on a joint work with Michal Fečkan and Natalia Dilna.

Schrödinger Equations and Quantum Models on Unitary Lattices

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We consider diagonalization processes for determining the energy spectrum of difference potentials in discrete Schrödinger quantum models. To do so, the Schrödinger equation is discretized on a grid which is a mixed version of an equidistant lattice and an adaptive linear grid – it is referred to by the name unitary lattice. Factorizations of the Schrödinger resp. Hamilton operators are derived and compared with the continuum situation. The arising spectral problems for these operators are addressed by using the theory of bilateral Jacobi operators in weighted $l^2(\mathbb{Z})$–spaces.

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Interrelation of Solutions of the Second Order Ordinary Differential Equations

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Solutions $x(t)$ of nonlinear differential equations of the type $x'' = f(t, x, x')$ are classified by the oscillatory behaviour of the respective equations of variations $y'' = f_x(t, x(t), x'(t))y + f_{x'}(t, x(t), x'(t))y'$. The type of a solution $x(t)$ depends on the number of zeros of the respective $y(t)$. If equations and solutions are given in a finite time interval $[a, b]$, is the behaviour of two solutions of a nonlinear equation arbitrary if their properties accodingly to classification are similar? If types of solutions of a nonlinear equation are known what are the restrictions on interrelation of these solutions? These and similar issues are considered in the talk. Consequences for the theory of boundary value problems are discussed.

New Results for Three-term Volterra Equations

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In this talk, using the fixed point theorem, we discuss the boundedness and stability of the zero solution of the discrete three-term Volterra equation

$$x(n+1) = a(n) + b(n)x(n) + \sum_{i=0}^{n} K(n;i)x(i).$$

Next, an asymptotic equivalence of some solution and the given sequence is considered. Necessary conditions for the existence of a periodic solution of the above equation are also presented.

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Solvability of Fučík Type BVP with Nonlocal Boundary Condition[1]

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The existence results are established for the nonlocal boundary value problem

$$x'' = -\mu x^+ + \lambda x^- + h(t,x,x') \quad ax(0) + bx'(0) = 0, \quad \int\limits_0^1 x(s)ds = 0$$

provided that $h(t,x,x')$ is continuous and Lipschitzian in x and x', but a, $b \in R$. The results are based on the study of spectrum for the problem

$$x'' = -\mu x^+ + \lambda x^- \quad ax(0) + bx'(0) = 0, \quad \int\limits_0^1 x(s)ds = 0.$$

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Controllability of Interconnected Evolution Equations with Unbounded Controls

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Let $V_1 \subset X_1 \subset V_1', V_2 \subset X_2 \subset V_2'$ be Hilbert spaces with continuous dense injections [1]. Consider the control evolution equation

$$\dot{x}_1(t) = A_1 x_1(t) + b_1 v(t),\ 0 \le t < +\infty, x_1(0) = x_1^0, x_1(t), x_1^0 \in X_1, b_1 \in V_1', v(t) \in \mathbf{R}, \tag{1.1}$$

where $v(t) = (c, x_2(t)), c \in X_2$ and $x_2(t)$ is a mild solution of the control evolution equation

$$\dot{x}_2(t) = A_2 x_2(t) + b_2 u(t),\ 0 \le t < +\infty, x_2(0) = x_2^0, x_2(t), x_2^0 \in X_2, b_2 \in V_2', v(t) \in \mathbf{R}, \tag{1.2}$$

Here the linear operators A_1 and A_2 generate strongly continuous C_0-semigroup $S_1(t)$ in X_1 and $S_2(t)$ in X_2 correspondingly.
Definition. Interconnected system (1.1)–(1.2) is said to be exact null-controllable on $[0, t_1]$ if for each $x_1^0 \in X_1$ there exists a square integrable control $u(\cdot) \in L_2[0, t_1]$ such that the a mild solution $x\left(t, x_{1,}^0 v(\cdot)\right)$ of equation (1.1) with a control $v(t)$, defined by $v(t) = (c, x_2(t))$, satisfies the condition $x_1\left(t_1, x_{1,}^0 v(\cdot)\right) = 0$.
Exact null-controllability conditions for linear control system consisting of two serially connected abstract control evolution equations (1.1)-(1.2) are presented. Applications to interconnected evolution eqiations describing boundaty control problems are considered.

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Integral Conditions for the Existence of Generalized Dichotomies in Banach Spaces

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In the general context of evolution operators, we obtain necessary and sufficient conditions for the existence of a notion of dichotomy which is both nonuniform and not necessarily exponential. Our main result is a type of Datko theorem for our generalized notion of dichotomy. We emphasize that this type of dichotomy includes as very particular cases the notions of nonuniform exponential dichotomy and nonuniform polynomial dichotomy and thus our theorems extend previous results. Additionally, we discuss some examples that illustrate our results.

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Chronotaxic Systems: A Class of Non-Autonomous Systems, with Time-Dependent Frequencies, Which Resist Continuous Perturbations

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Systems with time-dependent dynamics abound in nature – from astrodynamics or celestial mechanics, through ocean circulation, blood circulation in mammals, the dynamics of ionic concentration across the membrane of a living biological cell, to protein dynamics. However, when treated via an inverse approach – based on recorded data – their complex, but deterministic and non-autonomous, dynamics can often be misinterpreted as stochastic or chaotic. In this presentation we will introduce a special class of dynamical systems, where, despite the complex and stochastic-like dynamics, it is still possible to identify non-autonomous deterministic component of the dynamics. This class of systems is characterised by dynamics ordered in time and has been named chronotaxic (from *chronos* – time and *taxis* – order). Their basic properties are: (i) they have stable but time-varying characteristic frequencies, (ii) they are able to keep their complex dynamics stable against external perturbations, and (iii) they possess a time-dependent point attractor (driven steady state). We will elaborate the basic properties of chronotaxic systems and illustrate their application to a range of biological systems – from the cell, to the cardiovascular system and the brain.

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Discrete-Space Partial Differential Equation

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In this talk we consider a class of partial dynamic equations on domains with discrete-space and discrete/continuous/time scale time. Studying diffusion-type equations

$$u^{\Delta_t}(x,t) = au(x+1,t) + bu(x,t) + cu(x-1,t), \quad x \in \mathbb{Z},\ t \in \mathbb{T},$$

and its generalization

$$u^{\Delta_t}(x,t) = \sum_{i=-m}^{m} a_i u(x+i,t), \quad x \in \mathbb{Z}, \quad t \in \mathbb{T},$$

we observe that many properties of these equations (sign, symmetry and space-sum preservations, maximum principles, etc.) depend strongly on the underlying time structure. We derive continuous dependence on parameters and time scales. We discuss direct consequences for stochastic processes (Poisson-Bernoulli processes, random walks, general heterogenous processes...).

This is a joint work with Antonín Slavík (Charles University in Prague).

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Multi-Parameter Laser Modes in Paraxial Optics

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We study multi-parameter solutions of the inhomogeneous paraxial wave equation in a linear and quadratic approximation which include oscillating laser beams in a parabolic waveguide, spiral light beams, and other important families of propagation-invariant laser modes in weakly varying media. A "smart" lens design and a similar effect of superfocusing of particle beams in a thin monocrystal film are also discussed. In the supplementary electronic material, we provide a computer algebra verification of the results presented here, and of some related mathematical tools that were stated without proofs in the literature.

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Baker-Campell-Hausdorff Formula in the Hilbert Space Enviroment

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The Baker-Campell-Hausdorff formula in its shortest, terminating form looks like

$$\exp[A+B] = \exp[\frac{-|c|^2}{2}]\exp[A]\exp[B] \tag{1}$$

where the "operators" A and B satisfy $[A,B]=cI$, c a complex number. Formula (1) as well as its immediate consequence

$$\exp[A]\exp[B] = \exp[B]\exp[A] \tag{2}$$

sit in quantum optics, see for example [1, p. 519], but also mathematically, in its full form, in Lie group/algebra framework, see [2, Chapter 3] for a very cautious approach. I intend to show what survives of (1) and (2) if all that is settled in the Hilbert space environment.

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Principal and Antiprincipal Solutions of Linear Hamiltonian Systems

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In this talk we study the existence and properties of principal and antiprincipal solutions at infinity for possibly abnormal linear Hamiltonian systems. We show that the principal and antiprincipal solutions can be classified according to the rank of their first component and that they exist for any rank in the range between explicitly given minimal and maximal values. We also establish a limit characterization of the principal solutions by showing that they are the smallest ones at infinity when they are compared with the antiprincipal solutions. These are generalizations of the classical results of W. T. Reid, P. Hartman, or W. A. Coppel for controllable linear Hamiltonian systems. We illustrate our new theory by several examples.

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Estimates for the First Eigenvalue of a Sturm–Liouville Problem

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Consider the Sturm–Liouville problem

$$y'' - Q(x)y + \lambda y = 0, \quad x \in (0,1), \tag{1}$$

$$y(0) = y(1) = 0, \tag{2}$$

where Q belongs to the set $T_{\alpha,\beta,\gamma}$ of all real–valued locally integrable functions on $(0,1)$ with non–negative values such that the following integral condition holds:

$$\int_0^1 x^\alpha (1-x)^\beta Q^\gamma(x) dx = 1, \quad \alpha, \beta, \gamma \in \mathbb{R}, \;\; \gamma \neq 0. \tag{3}$$

We study the dependence of the first eigenvalue λ_1 on the potential Q under different values of parameters α, β, γ. For an arbitrary function $Q \in T_{\alpha,\beta,\gamma}$ consider the subspace H_Q of $H_0^1(0,1)$ with the norm

$$\|y\|^2_{H_Q} = \int_0^1 \left(y'^2 + Q(x) y^2 \right) dx.$$

For the first eigenvalue $\lambda_1(Q)$ of problem (1) – (2) it is proved (see [1]) that

$$\lambda_1(Q) = \inf_{y \in H_Q \setminus \{0\}} \frac{\int_0^1 \left(y'^2 + Q(x) y^2 \right) dx}{\int_0^1 y^2 dx}.$$

We give some estimates for $m_{\alpha,\beta,\gamma} = \inf\limits_{Q \in T_{\alpha,\beta,\gamma}} \lambda_1(Q)$, $M_{\alpha,\beta,\gamma} = \sup\limits_{Q \in T_{\alpha,\beta,\gamma}} \lambda_1(Q)$.

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Combinatorial Models for Spaces of Cubic Polynomials

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This is a joint work with Alexander Blokh, Lex Oversteegen, and Ross Ptacek.

Let $\mathbb{D}$ be the open disk $\{z \in \mathbb{C} \,|\, |z| < 1\}$ in the plane, $\overline{\mathbb{D}}$ be its closure, and $\mathbb{S}$ be its boundary circle. Let P be a polynomial of degree d with connected Julia set $J(P)$. We write Φ_P for the conformal isomorphism between $\mathbb{C} \setminus \overline{\mathbb{D}}$ and the complement U of the filled Julia set $K(P)$ asymptotic to the identity at infinity. By a theorem of Carathéodory, if $J(P)$ is locally connected, then Φ_P can be extended to a continuous map $\overline{\Phi}_P : \mathbb{C} \setminus \mathbb{D} \to \overline{U}$, under which $\mathbb{S}$ maps onto $J(P)$. Define the *lamination generated by* P as the equivalence relation $\sim_P$ on $\mathbb{S}$ identifying points of $\mathbb{S}$ if and only if $\overline{\Phi}_P$ sends them to the same point of $J(P)$.

By Thurston [5], the map P restricted to its locally connected Julia set $J(P)$ is topologically conjugate to a self-mapping $f_{\sim_P}$ of the quotient space $\mathbb{S}/\sim_P = J_{\sim_P}$ induced by $z^d|_{\mathbb{S}} = \sigma_d$; denote this conjugacy by $\Psi_P : J(P) \to J_{\sim_P}$. The mapping $f_{\sim_P}$ is called a *topological polynomial*. The quotient map of $\mathbb{S}$ onto $\mathbb{S}/\sim_P$ is denoted by $\pi_{\sim_P}$. Given a point $z \in J(P)$, we let $G_P(z) = G(z)$ denote the convex hull of the set $\pi^{-1}_{\sim_P}(\Psi_P(z))$. In other words, we represent z by the point $\Psi_P(z)$ of the model topological Julia set $J_{\sim_P}$ and then take all angles associated with $\Psi_P(z)$ in the sense of the lamination $\sim_P$. By [5], for two points z and w, the sets $G(z)$ and $G(w)$ either coincide or are disjoint.

The *geolamination* (from *geodesic* or *geometric lamination*) of P is the collection of chords, each of which is an edge of the convex hull of a $\sim_P$-class. Geolaminations geometrically interpret and "topologize" laminations, reflecting limit transitions among them. Both laminations and their geolaminations can be defined intrinsically (without polynomials). Then some geolaminations will not directly correspond to an equivalence relation on $\mathbb{S}$ but the family of all geolaminations will be closed. This allows one to work with limits of geolaminations and limits of polynomials (which might have non-locally connected Julia sets).

Thurston [5] models polynomials by their geolaminations, and families of quadratic polynomials by families of quadratic geolaminations. He "tags" quadratic geolaminations with their *minors* which form the *quadratic minor geolamination* QML and generate the corresponding lamination $\sim_{\mathrm{QML}}$. The quotient space $\mathbb{S}/\sim_{\mathrm{QML}}$ models the boundary of the Mandelbrot set $\mathcal{M}_2$ (this is the set of all parameters c such that polynomials z^2+c have connected Julia set; it is also called the *quadratic connected locus*). The induced quotient space of $\overline{\mathbb{D}}$ serves as a model for $\mathcal{M}_2$. Conjecturally, it is homeomorphic to $\mathcal{M}_2$.

Call a polynomial with connected Julia set *dendritic* if all its periodic points are repelling. If P is dendritic then the assumption that $J(P)$ is locally connected can be dropped. Indeed, by Kiwi [3], for any dendritic polynomial P there is a lamination $\sim_P$ such that there exists a *monotone semi-conjugacy* Ψ_P between $P|_{J(P)}$ and the topological polynomial $f_{\sim_P}$. Thus the sets $G_P(z) = \pi^{-1}_{\sim_P}(\Psi_P(z))$ are well-defined for every dendritic polynomial P and every point $z \in J(P)$. As we will see, these nice properties of *individual* dendritic polynomials result in nice properties of *families* of cubic dendritic polynomials.

Let $\mathcal{D}_2 \subset \mathcal{M}_2$ be the set of all parameters $c \in \mathcal{M}_2$ such that the polynomial $P_c(z) = z^2+c$ is dendritic. Set $H_c = G_{P_c}(c)$, and let $\mathcal{H}$ stand for the collection of all sets H_c, $c \in \mathcal{D}_2$. We denote the union $\bigcup_{c\in\mathcal{D}_2} H_c$ by $\mathcal{H}^+$ (in what follows, for any collection $\mathcal{A}$ of sets, we write $\mathcal{A}^+$ for the union of all sets in $\mathcal{A}$). By a part of a major result of [5], for two parameter values c, $c' \in \mathcal{D}_2$, the sets H_c and $H_{c'}$ are either disjoint or equal. Moreover, the mapping $c \mapsto H_c$ from $\mathcal{D}_2$ to $\mathcal{H}$ is upper semicontinuous. The set $\mathcal{D}_2$ (or, equivalently, the set of all dendritic quadratic polynomials defined up to a Moebius change of coordinates) projects continuously onto the quotient space of $\mathcal{H}^+$ defined by the partition of $\mathcal{H}^+$ into sets H_c with $c \in \mathcal{D}_2$.

We propose a related model for the space $\mathcal{MD}_3$ of *marked dendritic* cubic polynomials (P, c_1, c_2) with connected Julia set (c_1, c_2 are the critical points of P). Define the *co-critical* point associated to a critical point τ of P as the only point $\tau^* \neq \tau$ such that $P(\tau^*) = P(\tau)$ (if critical points of P are distinct) or τ itself (if P has a unique critical point in the the plane). Then, with every marked dendritic cubic polynomial (P, c_1, c_2), we associate the corresponding *mixed tag* $\theta(P, c_1, c_2) = G(c_1^*) \times G(P(c_2)) \subset \overline{\mathbb{D}} \times \overline{\mathbb{D}}$. This defines the mixed tag $\theta(P, c_1, c_2)$ for *all marked dendritic cubic polynomials*.

Theorem. *Mixed tags of elements in* $\mathcal{MD}_3$ *are disjoint or coincide so that sets* $\theta(P, c_1, c_2)$ *form a partition of the set* $\theta(\mathcal{MD}_3)^+ \subset \overline{\mathbb{D}} \times \overline{\mathbb{D}}$ *and generate the corresponding quotient space of* $\theta(\mathcal{MD}_3)^+$ *denoted by* MT_3. *Then* MT_3 *is a separable metric space and the map* $\theta : \mathcal{MD}_3 \to \mathrm{MT}_3$ *is continuous.*

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Nonsymmetric and Symmetric Fractional Calculi on Time Scales

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We introduce a nabla, a delta, and a symmetric fractional calculus on arbitrary nonempty closed subsets of the real numbers. These fractional calculi provide a study of differentiation and integration of noninteger order on discrete, continuous, and hybrid settings. Main properties of the new fractional operators are investigated, and some fundamental results presented, illustrating the interplay between discrete and continuous behaviors.

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Optimal Control Problems for Stochastic Differential Equations with Infinite Markov Jumps in Infinite Dimensions

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In this talk we are concerned with infinite horizon linear-quadratic (LQ) problems for infinite-dimensional stochastic differential equations (SDEs) affected simultaneous by countably infinite Markov jumps (MJs) and multiplicative noise (MN). We know that SDEs with Markovian switching can model many physical systems which experience abrupt changes in their dynamics. Without being exhaustive, we mention here the manufacturing systems, the power systems or the telecommunication systems which frequently suffer unpredictable structural changes caused by failures or repairs, connections or disconnections of the subsystems [1]. New applications of SDEs with countably infinite MJs arise in the field of modern queuing network theory, population and epidemic modeling or capital and asset price modeling, because these areas generate typical examples of Markov processes with countably infinite state space (we call infinite Markov processes). We recall here that the countable state models are usually used for systems which upper limit of the states values cannot be specified clearly. In this case the dynamic of an infinite Markov process should make the large state values of a system rare than if we use an upper limit to prevent them. The main difficulty in solving the proposed LQ optimal control problems is to work with infinite-dimensional control systems which switch from one mode to another according to the law of an infinite Markov chain. Our task is to minimize an infinite horizon quadratic cost functional over two different sets of admissible controls. Adopting the methods used in [2] for finite-dimensional LSDEs with MN and finite MJs, we shall see that, depending on the class of admissible controls, the optimal control is obtained either with the stabilizing solution or with the minimal solution of a corresponding system of generalized Riccati differential equations (GRDEs). In the infinite dimensional framework these GRDEs, which properties were recently investigated in [3], [4], are more complicated being defined on ordered Banach spaces of sequences of operators.

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Discrete and Continual Calderon - Zygmund Operators

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Such operators arise in a lot of applied problems, and the studying their properties is of a special interest. The simple Calderon-Zygmund operator

$$v.p. \int_{\mathbf{R}^m} K(x-y)u(y), \quad x \in \mathbf{R}^m,$$

is like a convolution, and this fact permits to obtain some interesting its properties.

If we consider a discrete variant of such operator on integer lattice $\mathbf{Z}^m$ or on the discrete half-space, then we'll find many common properties of discrete and continual operators. More general case is related to the Calderon - Zygmund operator with variable kernel

$$v.p. \int_{D} K(x, x-y)u(y), \quad x \in \mathbf{R}^m, \quad x \in D,$$

where $D \subset \mathbf{R}^m$, and its discrete analogue also has many common properties with continual operator.

This is joint work with A.V. Vasilyev.

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Existence of Chaos in Plane $\mathbb{R}^2$ and its Application in Macroeconomics

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We research the Devaney, Li-Yorke and distributional chaos in plane $\mathbb{R}^2$ which exists in the continuous dynamical system generated by Euler equation branching. Euler equation branching is a type of differential inclusion $\dot{x} \in \{f(x), g(x)\}$, where $f, g : X \subset \mathbb{R}^n \to \mathbb{R}^n$ are continuous and $f(x) \neq g(x)$ in every point $x \in X$. In [2] there is defined so-called chaotic set in the plane $\mathbb{R}^2$ which existence leads to an existence of Devaney, Li-Yorke and distributional chaos. We show that two hyperbolic singular points of both branches not lying in the same point in $\mathbb{R}^2$ always admit so-called chaotic sets. Then we describe the set of solutions of Euler equation branching which ensures Devaney, Li-Yorke and distributional chaos in so-called chaotic sets in the situation of two hyperbolic singular points.
In the second part we present own new overall macroeconomic equilibrium IS-LM/QY-ML model. This model is based on the fundamental macroeconomic equilibrium IS-LM model. We model an inflation effect, an endogenous money supply and an influences of the economic cycle on the economy in addition to the original model. We research the dynamical behaviour of this new model. The chaos seems to be natural part of macroeconomic systems under the view of this new model.

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Discussion on Polynomials Having Polynomial Iterative Roots

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In this work we discuss polynomial mappings which have iterative roots of polynomial form. We apply the computer algebra system *Singular* to decompose algebraic varieties and finally find a condition under which polynomial functions have quadratic iterative roots of quadratic polynomial form. This condition is equivalent to but simpler than Schweizer and Sklar's and more convenient than Strycharz-Szemberg and Szemberg's. We further find all polynomial functions which have cubic iterative roots of quadratic polynomial form and compute all those iterative roots. Moreover, we find all 2-dimensional homogeneous polynomial mappings of degree 2 which have iterative roots of polynomial form and obtain expressions of some iterative roots. This is a joint work by Zhiheng Yu, Lu Yang and Weinian Zhang.

Lower Bounds for Eigenvalues of Planar Hamiltonian Systems

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We establish lower bounds estimates for eigenvalues of first-order planar Hamiltonian systems. As a special case a lower estimate on eigenvalues of linear equations is also obtained. The method of proof requires suitable use of Lyapunov-type inequalities.

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Local Bifurcations of the Enzyme-Catalyzed Reactions Comprising a Branched Network

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In this paper an enzyme-catalyzed reaction system with four parameters a, b, c, κ is discussed. The system can be reduced to a quartic polynomial differential system with four parameters, which makes not only large-scale polynomials in computation but.. also difficulties of semi-algebraic systems restricted to subsets of special biological sense, none of which is closed under operations of the polynomial ring. We give parameter conditions for exact number of equilibria and for all co-dimension 1 bifurcations such as saddle-node bifurcation, transcritical bifurcation and pitchfork bifurcation. Hopf bifurcations are discussed by computing varieties of Lyapunov quantities restricted by some inequalities for biological requirements. The order of weak focus is proved to be at most 2 and conditions are given for exact order. Finally, our results are illustrated by numerical simulations.

Roughness of Tempered Exponential Dichotomies for Random Difference Equations

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This work is concerning the roughness of tempered exponential dichotomies for linear random dynamical systems in Banach spaces. Such a dichotomy has a tempered bound and describes nonuniform hyperbolicity. The roughness is proved without assuming their invertibility and the integrability condition of the Multiplicative Ergodic Theorem. An explicit bound is given for the linear perturbation such that the dichotomy is persistent. Explicit forms are obtained for the exponent and the bound of tempered exponential dichotomy of the perturbed random system in terms of the original ones and the perturbations. This is a joint work by Linfeng Zhou, Kening Lu and Weinian Zhang.

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Boundedness of Nonoscillatory Solutions of Three–Dimensional Nonlinear Difference Systems

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We consider a three–dimensional nonlinear difference system with deviating arguments of the following form

$$\begin{cases} \Delta(x_n + px_{n-\tau}) &= a_n f(y_{n-l}) \\ \Delta y_n &= b_n g(w_{n-m}), \\ \Delta w_n &= \delta c_n h(x_{n-k}) \end{cases}$$

where the first equation of the system is a neutral type difference equation. Firstly, the classification of nonoscillatory solutions of the considered system are presented. Next, we present the sufficient conditions for boundedness of a nonoscillatory solution. The obtained results are illustrated by examples.

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Fractal Analysis of Unit-time Map and Cyclicity of Nilpotent Singularities of Planar Vector Fields

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This article shows how fractal analysis of the unit-time map can be used in studying the cyclicity problem of nilpotent singularities. We study fractal properties such as box dimension and ε-neighbourhood of discrete orbits generated by the unit-time map. In the case of bifurcations of non-hyperbolic singularities such as saddle-node or Hopf-Takens bifurcation, there is already known connection between the multiplicity of singularity and the box dimension of the unit-time map or Poincaré map. In this article we study how the box dimension and ε-neighbourhood of discrete orbits generated by the unit-time map near nilpotent singularities are connected to the known bounds for local cyclicity of singularities. In this analysis, we use the restriction of the unit-time map on the characteristic curves or separatrices, depending on the type of singularity. Main nilpotent singularities which are studied here are nilpotent node, focus and cusp. Moreover, we study fractal properties of the unit-time map for nilpotent singularities at infinity.

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