Technical work in WP2 and WP5

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WP2 Work Plan (Task 2.4)

- Analysis of applicability and estimation of computational complexity of different uncertainty management strategies for SoS. \((M15 - M20)\)
- Development of scenario-based robust dynamic management for SoS. \((M18 - M24)\)
- Development of a stochastic methodology for the coordination of systems using SoS model subject to Gaussian distribution of uncertainties and disturbances. \((M21 - M27)\)
- Development of a stochastic approach to the coordination of systems using SoS model subject to uncertainties and disturbances with arbitrary distributions, using both explicit and scenario-based approach. \((M24 - M30)\)
- Development of methods for representation of the resulting SoS performance in stochastic sense, for merging with its neighboring systems. \((M30 - M33)\)
WP5 Work Plan (Task 5.2)

• Mathematical modeling of the case studies. (M07-M18)
• Implementation of the models. (M09-M18)
• Derivation of simplified models and implementation in MATLAB. (M12-M22)
• Documentation. (M22-M24)
Motivation
Simulation framework – a quick reminder

**Requirements:**
- MATLAB (tested on version 8.3.0.532, R2014a)
- Windows (tested on Windows 8.1), should work on Mac OS X, Linux (e.g. Ubuntu), and older Windows (7, XP)
- SimPowerSystems - standard Simulink toolbox for simulation of electrical power systems
- For the OPF routines additional MATLAB packages are needed:
  - YALMIP
  - A dedicated solver (e.g. SeDuMi, SDPT3, Mosek, CPLEX)

**Advantages:**
- Out-of-box integration with MATLAB
- Dynamic simulation of power system (phasor, continuous, discrete)
- Many dynamic models already implemented in the toolbox
Simulation framework – key features

• Automatic generation of simulation models
  – The development of (large-scale) power network models is very tedious, time-consuming and prone to human error
  – A network is already described in a structured file format
  – Write MATLAB script/function that can interpret a network description and automate the creation of the simulation model

• Automatic initialization
  – Initialization of the dynamic simulation model of the network
  – Use OPF solution as an initial steady state operating point

• Full advantage of SimPowerSystems model libraries
Constraints and target (1)

- **Objective:** minimize total active power losses over a prediction horizon $N$
  \[
  \sum_{t=0}^{N} \sum_{i \in \mathcal{V}} P_{i,t}^I
  \]

- **Power balance constraints**
  \[
  P_{i,t}^I = P_{i,t}^G - P_{i,t}^D = \sum_{j \in \mathcal{N}(i)} \delta_{i,j,t} P_{i,j,t}, \forall i \in \mathcal{V}
  \]
  \[
  Q_{i,t}^I = Q_{i,t}^G - Q_{i,t}^D = \sum_{j \in \mathcal{N}(i)} \delta_{i,j,t} Q_{i,j,t}, \forall i \in \mathcal{V}
  \]
  \[
  P_{i,j,t} = g_{ij} V_{i,t}^2 - V_{i,t} V_{j,t} (g_{ij} \cos(\theta_{i,j,t}) + b_{ij} \sin(\theta_{i,j,t}))
  \]
  \[
  Q_{i,j,t} = -b_{ij} V_{i,t}^2 + V_{i,t} V_{j,t} (b_{ij} \cos(\theta_{i,j,t}) - g_{ij} \sin(\theta_{i,j,t}))
  \]
Constraints and target (2)

• Voltage constraints
  \[ V \leq V_{i,t} \leq \bar{V}, \forall i \in \mathcal{V} \]

• Current constraint
  \[ I_{i,j,t}^2 = \delta_{i,j,t} \left( g_{i,j}^2 + b_{i,j}^2 \right) \left( V_{i,t}^2 + V_{j,t}^2 - 2V_{i,t}V_{j,t} \cos(\theta_{i,j}) \right) \leq \bar{I}_{i,j}^2 \]

• Battery storage constraints
  \[ x_{i,t+1}^{BAT} = x_{i,t}^{BAT} - \eta_i P_{i,t}^{BAT} \]
  \[ \eta_i = \begin{cases} 
    \eta_i^c, & P_{i,t}^{BAT} < 0 \text{ (charging)} \\
    1/\eta_i^d, & \text{otherwise (discharging)}
  \end{cases} \]
Constraints and target (3)

- Battery storage constraints (cont’d)
  \[ X_i \leq x_{i,t}^{BAT} \leq \bar{X}_i \]
  \[ \underline{P}_i^{BAT} \leq P_{i,t}^{BAT} \leq \bar{P}_i^{BAT} \]
  \[ \underline{Q}_i^{BAT} \leq Q_{i,t}^{BAT} \leq \bar{Q}_i^{BAT} \]

- Generator constraints
  \[ x_{i,t+1}^G = A x_{i,t}^G + B u_{i,t}^G \]
  \[ y_{i,t}^G = C x_{i,t}^G + D u_{i,t}^G \]
  \[ \underline{P}_i^G \leq P_{i,t}^G \leq \bar{P}_i^G \]
  \[ \underline{Q}_i^G \leq Q_{i,t}^G \leq \bar{Q}_i^G \]
Simulation scenarios

• Line fault
• Generator outage
• Major change of the overall system demand
Microgrid
Objective function and constraints

- Economic criterion

\[
J(u, x_0, c, d) = -c_N x_N + \sum_{k=0}^{N-1} c_k P_k^G \Delta T
\]

\[
c_N = \frac{p_{pct}}{100} \max_k c_k , \quad t \leq k \leq t + N - 1
\]

- Constraints

\[
x_{\text{min}} \leq x_k \leq x_{\text{max}}
\]

\[
u_{\text{min}} \leq u_k \leq u_{\text{max}}
\]

\[
P^G_{\text{min}} \leq P_k^G \leq P^G_{\text{max}}
\]

- LP formulation

\[
\min_u f^T u + \text{const}
\]

\[
s.t. E_x x_0 + E_u u + E_d d \leq g
\]
Penalization of residual storages state

Revenue at the end of the one-year period [EUR]

Share of the maximum price value over the prediction horizon $\rho_{\text{ct}}$ [%]

- $N=3$
- $N=6$
- $N=18$
- $N=24$
Power distribution system

• Sources of uncertainty:
  – Power demand prediction
  – Uncontrollable generation prediction (e.g. wind, sun)

• Robust Optimal Power Flow (ROPF)
  – Compute optimal power references such that constraints are robustly satisfied for all possible realizations of uncertainty

• Objective
  – E.g. nominal objective (total system losses)

• Constraints
  – Must be satisfied for every realization of uncertainty
Exponential growth of num. of constraints

- Let $P_i^D = P_{i0}^D + \Delta P_i^D$, $\Delta P_i^D \in \left[\Delta P_i^D, \Delta P_i^D\right]$

- Then $P^D = [P_1^D \quad P_2^D \quad ... \quad P_n^D]^T \in \mathcal{W}_P^D$
  - $\mathcal{W}_P^D = \text{co}\{P_1^D, P_2^D, ..., P_m^D\}$, where $P_i^D$ is one extreme realization of demand uncertainty

- The same goes for reactive power demand $Q^D$

- Constraints must be satisfied for every combination of extreme realizations $P_i^D$ and $Q_i^D$!

- The number of constraints grows exponentially with number of nodes in the network!
Tube MPC (1)

• System dynamics:
  \[ x^+ = Ax + Bu + w, \quad (x, u) \in \mathbb{X} \times \mathbb{U}, \quad w \in \mathbb{W} \]

• Idea: Separate control into two parts
  1. A portion that steers the nominal system to the origin
     \[ z^+ = Az + Bv \]
  2. A portion that compensates for deviations from the nominal system
     \[ e^+ = (A + BK)e + w \]

• The linear feedback controller \( K \) is fixed offline (such that \( A + BK \) is stable)

• Keep “real” trajectory close to the nominal
  \[ u_i = K(x_i - z_i) + v_i \]
Tube MPC (2)

- Bound maximum error (how far is the "real" trajectory from the nominal)
  - Compute **minimum** robust positive invariant set $\mathcal{E}$
  - $x_i \in z_i \oplus \mathcal{E}$

- Compute tightened constraints on nominal system
  - $z_i \in \mathcal{X} \ominus \mathcal{E}$
  - $v_i \in \mathcal{U} \ominus K\mathcal{E}$

- Formulate as convex optimization problem
  - Optimize the nominal system with tightened constraints
  - The cost is with respect to tube centers $z_i$
  - Choose terminal set and terminal objective function to ensure robust feasibility and stability
Scenario-based approach (1)

- Taking into account every possible combination of extreme uncertainty realization leads to an untractable optimization problem
- Scenario approach: take into account only \( N \) uncertainty realization scenarios (instead of all possible combinations of extreme uncertainty realizations)
- Constraint violation is tolerated, but with some probability level \( \varepsilon \)
  - E.g instead \( x \in X \) we have \( \mathbb{P}\{x \in X\} \geq 1 - \varepsilon \)
- Additionally, we can discard \( k \) out of \( N \) scenarios in order to improve the objective value
Scenario-based approach (2)

• Theoretical guarantees that the solution obtained by inspecting $N$ scenarios only is a feasible solution for the chance-constrained optimization problem with high probability $1 - \beta$, provided that $N$ and $k$ fulfill the following condition:

$$\binom{k+d-1}{k} \sum_{i=0}^{k+d-1} \binom{N}{i} \varepsilon^i (1 - \varepsilon)^{N-i} \leq \beta$$

where $d$ is the number of optimization variables.

• Choose $N$ within the computational limit of the used solver, $\varepsilon$ according to the acceptable level of risk, $\beta$ small enough to be negligible (e.g. $\beta = 10^{-10}$), and compute the largest $k$ number of scenarios that can be discarded
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