Production of photons to the order $\alpha$ in out of equilibrium QGP

Domagoj Kuić
Faculty of natural sciences, mathematics and kinesiology, University of Split

Prepared for ”Hot matter and gauge field theories”, 30. 8. - 2. 9. 2007, Rab, Croatia
Heavy ion collisions at ultrarelativistic energies - possibility to examine the fundamental properties of QCD:

- asymptotic freedom

- deconfinement

- quarks and gluons - quasifree particles in experimental conditions (?)

⇒ Model of quark-gluon plasma (QGP) is based on asymptotic freedom of QCD
QCD at high temperature and matter density:

- thermal field theory
- out of equilibrium field theory

Necessary for calculation of QGP properties - knowledge of the initial state

⇒ Parton cascade models:

- parton collisions lead to forming of hot plasma of quarks and gluons

- thermalisation after $0.1 - 1 \text{ fm/c}$ (?) and hydrodynamic expansion
Emission of hadron particles follows after rehadronisation and freeze-out.

Possible phase coexistence: QGP and hadronic phase (hadron gas, DCC).

Photons and leptons: emitted in all phases of the collision, leave the plasma without further interactions.

⇒ clean signals of QGP forming in experiments.
Photon production

Photon production in ultrarelativistic heavy ion collision - standard approach
- different contributions:

- photons emitted in hard parton interactions similar to QCD Compton scattering, annihilation, bremsstrahlung

- photons emitted in quark-gluon collisions in QGP

- photons emitted by hadronic particles ($\pi^0$, $\rho$, $\omega$, ...) after rehadronisation
Model of photon production in out of equilibrium field theory

- system is prepared at $t_i$ in initial state
- at $t_i = 0$ time evolution starts and ends at $t_f$

Time evolution defined along the contour $C = C_1 \cup C_2$ in complex plane
Time ordering $T_c$ is defined on the contour $C$ and equal to zero elsewhere

Averages of operators (density operator $\rho$)

$$\langle O(t) \rangle = \text{Tr} \rho O(t)$$

Two-point Green function

$$G^c(x, x') = -i \langle T_c \phi(x) \phi(x') \rangle$$

$G^c(x, x')$ for $t, t'$ on $C = C_1 \cup C_2$ connected with components of the Keldysh space $G_{ij}(x, x')$ ($i, j = 1, 2$)

Limit $t_i \to -\infty$, $t_f \to \infty \Rightarrow$ nontrivial connection with the Keldysh approach
Projected functions

Two-point function $G(x, y)$ with four-vector variables $x, y$, time components inside the interval $t_i < x_0, y_0 < \infty$

Wigner variables

$$X = \frac{x + y}{2}, \quad s = x - y$$

$$G(x, y) = G(X + \frac{s}{2}, X - \frac{s}{2})$$

Lower boundary for $x_0, y_0 \Rightarrow 0 < X_0$ and $-2X_0 < s_0 < 2X_0$

The values of $G(X + \frac{s}{2}, X - \frac{s}{2})$ for $(X, s)$ not satisfying $0 < X_0$ and $-2X_0 < s_0 < 2X_0$ are physically irrelevant
Definition of time ordering $T_c$ sets them to zero outside intervals $0 < X_0, -2X_0 < s_0 < 2X_0$

$G(X + \frac{s}{2}, X - \frac{s}{2})$ is written using the projection operator

$$G(X + \frac{s}{2}, X - \frac{s}{2}) = \theta(X_0)\theta(2X_0 - s_0)\theta(2X_0 + s_0)$$

$$\times \bar{G}(X + \frac{s}{2}, X - \frac{s}{2})$$

Outside the carrier of the projection operator values of $\bar{G}(X + \frac{s}{2}, X - \frac{s}{2})$ are arbitrary
Wigner transform of projected function - Fourier transform w.r.t. \((s_0, \vec{s})\)

\[
G(p_0, \vec{p}, X) = \int d^4s \, e^{i(p_0s_0 - \vec{p} \cdot \vec{s})} G(X + \frac{s}{2}, X - \frac{s}{2})
\]

\[
= \frac{1}{(2\pi)^4} \int d^4p \, e^{-i(p_0s_0 - \vec{p} \cdot \vec{s})} G(p_0, \vec{p}, X)
\]

Homogeneity in space coordinates excludes dependence on \(\vec{X}\)
Projection operator has a simple transform

\[ P_{X_0}(p_0, p'_0) \]

\[ = \frac{1}{2\pi} \theta(X_0) \int_{-2X_0}^{2X_0} ds_0 \ e^{is_0(p_0-p'_0)} \]

\[ = \frac{1}{\pi} \theta(X_0) \frac{\sin (2X_0(p_0 - p'_0))}{p_0 - p'_0} \]

Important property for energy conservation in the limit \( X_0 \to \infty \)

\[ \lim_{X_0 \to \infty} P_{X_0}(p_0, p'_0) = \delta(p_0 - p'_0) \]

For finite \( X_0 \) (finite time) \( \Rightarrow \) energy nonconservation
Function $\bar{G}(X + \frac{s}{2}, X - \frac{s}{2}) = \bar{G}(s_0, \vec{s})$ follows from the projected function in the limit $X_0 \to \infty$

$$\lim_{X_0 \to \infty} G(X + \frac{s}{2}, X - \frac{s}{2}) = \bar{G}(s_0, \vec{s})$$

Important property of projected functions $\Rightarrow$ transform of the projection operator induces $X_0$ dependence

$$G_{X_0}(p_0, \vec{p}) = [P_{X_0} G_{\infty}] (p_0, \vec{p})$$

$$= \int_{-\infty}^{\infty} dp_0' P_{X_0}(p_0, p_0') G_{\infty}(p_0', \vec{p})$$
Important examples of projected functions are retarded, advanced and Keldysh component of free propagators.

For further analysis analytic properties in the $X_0 \rightarrow \infty$ limit of Wigner transforms of projected functions (WTPF) are important.

We define the following properties corresponding to R (A) components:
(1) function of $p_0$ is analytic above (below) real axis
(2) function goes to zero as $|p_0|$ approaches infinity in the upper (lower) semiplane.
Convolution products of projected functions

\[ C = A_1 * A_2 * \ldots * A_{n-1} * A_n \]

For convolution products of \( n \) projected functions it is important that at least \( n - 1 \) functions satisfy assumptions (1) and (2).

Order is also important: the retarded functions should be on the right, the advanced on the left, and the functions neither advanced nor retarded in the middle.
If these conditions are fulfilled:

\[ C_{X_0}(p_0, \vec{p}) \]

\[ = \int dp_{0,1} P_{X_0}(p_0, p_{0,1}) \prod_{j=1}^{n} A_{j,\infty}(p_{0,1}, \vec{p}) \]

\[ \Rightarrow \] Convolution products of Wigner transforms of projected functions are Wigner transforms of projected functions (WTPF)

But propagators and self-energies in the Schwinger-Dyson equations appear in different order

\[ \Rightarrow \] terms that are not WTPF appear in Schwinger-Dyson equations
Schwinger-Dyson equations

Schwinger-Dyson equations for R, A and K components of the propagator

\[
G_R = G_R + iG_R \ast \Sigma_R \ast G_R
\]

\[
G_A = G_A + iG_A \ast \Sigma_A \ast G_A
\]

\[
G_K = G_K + iG_R \ast \Sigma_K \ast G_A
\]

\[
+ iG_K \ast \Sigma_A \ast G_A + iG_R \ast \Sigma_R \ast G_K
\]
Formal solutions for retarded and advanced component

\[ G_R = G_R \ast (1 - i\Sigma R \ast G_R)^{-1} \]

\[ G_A = G_A \ast (1 - i\Sigma A \ast G_A)^{-1} \]

R and A components of the resummed propagator are Wigner transform of projected functions (WTPF)

Keldysh component of the resummed propagator

\[ G_K = G_R \ast \left( h(p_0, \omega_p)(G_A^{-1} - G_R^{-1}) \right) \]

\[ + i\Sigma_K \ast G_A \]
Keldysh component of self-energy does not satisfy assumptions (1) and (2)

One-loop approximation to $\Sigma_K$ can be decomposed into parts satisfying (1) and (2) as retarded and advanced functions

$$\Sigma_K = -\Sigma_{K,R} + \Sigma_{K,A}$$

But Schwinger-Dyson equation for $K$ component of the propagator contains retarded components on the left from the advanced components

$\implies$ stepping out of the space of Wigner transforms of projected functions (WTPF)
Equal time two-point functions and observables

To study single-particle observables: reduction of two-point functions to equal time ($x_0 = y_0 = t \Rightarrow X_0 = t$, $s_0 = 0$) ⇒ it is obtained by inverse Wigner transform

$$G(t, 0, \vec{p}) = \frac{1}{2\pi} \int dp_0 G_{X_0=t}(p_0, \vec{p})$$

Average number of particles with impulse $\vec{p}$ is connected to equal time $K$ component of the propagator

$$\langle 2N_{\vec{p}}(t) + 1 \rangle = \frac{\omega_p}{2\pi} \int dp_0 G_{K,t}(p_0, \vec{p})$$
Other single-particle observables are generated with the help of $\langle N_{\vec{p}}(t) \rangle$

For projected functions and bare fields

$$\langle 2N_{\vec{p}}(t) + 1 \rangle = \frac{\omega_p}{2\pi} \int dp_0' G_{K,\infty}(p_0', \vec{p})$$

$$= 1 + 2f(\omega_p)$$

This is completely determined by its form in the $X_0 \rightarrow +\infty$ limit
Equal time $K$ component of the propagator in the single self-energy insertion approximation $G_K = G_K^0 + G_K^1 + \ldots$

$$\langle 2N_{\vec{p}}(t) + 1 \rangle$$

$$= \langle 2N_{\vec{p}}^0(t) + 1 \rangle + \langle 2N_{\vec{p}}^1(t) \rangle + \ldots$$

$$= 1 + 2f(\omega_p) + \frac{\omega}{2\pi} \int dp_0 G_{K,X_0}^1(p_0, \vec{p})$$

Time dependence of single-particle observables is described by equal time two-point functions

All terms coming from WTPF are constants in time

$\Rightarrow$ non-WTPF terms generate the time dependence
Number of photons in QGP

Average photon number with impulse \( \vec{p} \) produced in QGP

\[
\langle N_{\vec{p}}(t) \rangle = \frac{dN(t)}{d^3p d^3x} (2\pi)^3
\]

\[
= \frac{\omega_p}{4\pi} \int dp_0 [D_{t,K}(p_0, \vec{p}) - D_{0,K}(p_0, \vec{p})]
\]

Assumption: initial state (at \( t_i = 0 \)) contains no photons

\( \Rightarrow "\text{"prompt\"} \) photons leave the medium
Phase space photon number density

\[
\frac{dN(t)}{d^3p d^3x} = \frac{\langle N_{\vec{p}}(t) \rangle}{(2\pi)^3}
\]

\[
= -\frac{2}{\pi (2\pi)^3} \frac{p}{2} \left( \int_{-\infty}^{\infty} dp_0 \mathcal{P} \frac{\text{Im} \tilde{\Sigma}_{\infty, K, R}(p_0, \vec{p})}{(p_0^2 - \vec{p}^2)^2} \right)
\]

\[
\left[ 1 - \cos(p_0 - p)t + \frac{p - p_0}{p} \sin tp_0 \sin tp \right]
\]

\[
+ 2\pi \frac{1}{4p^3} \sin^2 tp \sum_{\lambda = \pm 1} \lambda \text{Re} \tilde{\Sigma}_{\infty, K, R}(\lambda p, \vec{p})
\]
Photon number density with vacuum contribution (dashed line) and photon number density without vacuum contribution (full line) vs. photon impulse $p$ at $t = 10$ fm/c. Parameter $T$ is equal 0.2 GeV. Quark masses ($u$ i $d$) are set equal to zero.
- photon number density is negative at small impulse $p \ll T$ (region where resummation is necessary - HTL)

- at large impulse total photon number and energy are infinite

This is a consequence of the choice of initial conditions: initial states are eigenstates on the Fock space of non-interacting hamiltonian

$\Rightarrow$ regularization is necessary
Regularization

Problem of initial conditions: in one-loop approximation (order $\alpha$ in coupling constant) total energy emitted through photon field is infinite

Without the formal solution, finite results at the order $\alpha$ can be achieved, by considering four basic types of QCD plasma
1. vacuum plasma with initial quark and antiquark distribution functions equal to zero \((f_+ = 0 \text{ and } f_- = 0)\)

2. quark plasma with \(f_+ \neq 0 \text{ and } f_- = 0\)

3. antiquark plasma with \(f_- \neq 0 \text{ and } f_+ = 0\)

4. quark-antiquark plasma with \(f_+ \neq 0 \text{ and } f_- \neq 0\)

For ”bare” initial conditions all four types of plasma emit infinite amount of energy at the order \(\alpha\) in the coupling constant
Had we prepared ’dressed’ initial conditions only quark-antiquark plasma should emit photons at the order $\alpha$ in the coupling constant.

Quark-antiquark plasma contains other three types of plasma and reduces on them as special cases.

By substracting these contributions to average photon number regularized expression is obtained which gives finite total energy

$$N_{f_+,f_-}\text{reg}(\vec{p}, t) = N_{f_+,f_-}(\vec{p}, t)$$

$$-N_{f_+,0}(\vec{p}, t) - N_{0,f_-}(\vec{p}, t) + N_{0,0}(\vec{p}, t)$$
Regularized phase space photon number density for parameter $T = 0.2$ GeV and $t = 10$ fm/c vs. photon impulse
- function is positive at small impulse
  \((p \ll T)\)

- function falls exponentially at large impulse

\(\Rightarrow\) regularization gives finite total emitted photon energy
Regularized phase space photon number density for parameter $T = 0.3$ GeV and $t = 10$ fm/c vs. photon impulse.
Photon number density at $p = 0.2$ GeV/c vs. time. Parameter $T$ is equal 0.2 GeV.
Photon number density at $p = 0.02$ GeV/c vs. time. Parameter $T$ is equal 0.2 GeV.
