Introduction to Generalized Parton Distributions, DVCS and DVMP

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Outline

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2. DVCS, DVMP, GPDs — theory
   - Deeply virtual Compton scattering (DVCS)
   - . . . , deeply virtual meson electroproduction (DVMP)
   - Generalized parton distributions (GPDs)

3. DVCS, DVMP, GPDs — phenomenology
   - Experimental status
   - Towards unravelling GPDs
   - Modeling venues
   - One example approach...

4. Summary
Resolving nucleon structure

SCATTERING

→ elastic \((e^- p \rightarrow e^- p)\) \}\{ exclusive

→ inelastic \((e^- p \rightarrow e^- \pi p)\)
\((e^- p \rightarrow e^- X)\) \}\{ inclusive
Resolving nucleon structure

SCATTERING

\[ \rightarrow \text{elastic} \quad (e^- p \rightarrow e^- p) \quad \} \quad \text{exclusive} \]
\[ \rightarrow \text{inelastic} \quad (e^- p \rightarrow e^- \pi p) \quad \} \quad \text{exclusive} \]
\[ (e^- p \rightarrow e^- X) \quad \} \quad \text{inclusive} \]

ELASTIC SCATTERING on a pointlike particle (s=1/2)

\[ \gamma^\mu \rightarrow iA \]
\[ \frac{d\sigma}{d\Omega_{lab}} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{E'}{E} \left\{ 1 \cos^2 \frac{\theta}{2} - \frac{1}{2} \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right\} \]
\[ \sim |A|^2 \]
ELASTIC SCATTERING on a composite particle

\[ F_1, F_2 \ldots \text{Dirac and Pauli form factors} \]

\[ G_E = F_1 + \frac{\kappa q^2}{4M^2} F_2 \]
\[ G_M = F_1 + \kappa F_2 \]

\[ G_E^p(0) = 1, \quad G_E^n(0) = 0 \]
\[ G_M^p(0) = 2.79, \quad G_M^n(0) = -1.91 \]

\[ F_1(q^2) \gamma^\mu + \frac{\kappa}{2M} F_2(q^2) i\sigma^{\mu\nu} q_\nu \rightarrow iA \]
ELASTIC SCATTERING on a composite particle

\[ F_1, F_2 \ldots \text{Dirac and Pauli form factors} \]

\[ G_E = F_1 + \frac{\kappa q^2}{4M^2} F_2 \quad G_E^p(0) = 1, \quad G_E^n(0) = 0 \]

\[ G_M = F_1 + \kappa F_2 \quad G_M^p(0) = 2.79, \quad G_M^n(0) = -1.91 \]

\[ F_1(q^2) \gamma^\mu + \frac{\kappa}{2M} F_2(q^2) i\sigma^{\mu\nu} q_\nu \quad \rightarrow \quad iA \]

\[ \frac{d\sigma}{d\Omega_{lab}} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{E'}{E} \left\{ K_2(q^2) \cos^2 \frac{\theta}{2} - K_1(q^2) \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right\}_{ep \rightarrow ep} \]

\[ \sim |A|^2 \quad \text{[Rosenbluth formula]} \]
INELASTIC INCLUSIVE SCATTERING

$q = (\nu, \bar{q})$

scalars often used:

$E', \theta$ (exp.)

$q^2, \nu = \frac{q \cdot p}{M} = E - E'$ (teor.)

$q^2, x = \frac{-q^2}{2q \cdot p}$ (teor.)
INELASTIC INCLUSIVE SCATTERING

\[ q = (\nu, \bar{q}) \]

Scalars often used:

\[ E', \theta \text{ (exp.)} \]

\[ q^2, \nu = \frac{q \cdot p}{M} = E - E' \text{ (teor.)} \]

\[ q^2, x = \frac{-q^2}{2q \cdot p} \text{ (teor.)} \]

\[ \mathcal{W}^{\mu \lambda} = -W_1 g^{\mu \lambda} + \frac{W_2}{M^2} p^{\mu} p^{\lambda} + \frac{W_4}{M^2} q^{\mu} q^{\lambda} + \frac{W_5}{M^2} (p^{\mu} q^{\lambda} + p^{\lambda} q^{\mu}) \]

\[ q_\mu \mathcal{W}^{\mu \lambda} = q_\lambda \mathcal{W}^{\mu \lambda} = 0 \]

\[ d\sigma \sim L_{\mu \lambda}^e \mathcal{W}^{\mu \lambda} \]

\[ \sim \left\{ W_2(q^2, \nu) \cos^2 \frac{\theta}{2} + 2W_1(q^2, \nu) \sin^2 \frac{\theta}{2} \right\}_{ep \rightarrow eX} \]

\[ W_1, W_2 \ldots \text{structure functions} \]
Bjorken limit:

\[ q^2 \rightarrow \infty \quad \text{and} \quad \nu \rightarrow \infty \]

\[ x = x_B = \frac{-q^2}{2q \cdot p} = \text{cte}. \]
DEEP INELASTIC SCATTERING

Bjorken limit:

\[ q^2 \to \infty \quad \text{and} \quad \nu \to \infty \]

\[ x = x_B = \frac{-q^2}{2q \cdot p} = \text{cte.} \]

\[ \to \text{sum of elastic } e^-\text{-parton scatterings} \]

structure functions:

\[ M \ W_1(q^2, x) \to F_1(x) \]

\[ -\frac{q^2}{2Mx} \ W_1(q^2, x) \to F_2(x) \]
SCALING VIOLATION IN DEEP INELASTIC SCATTERING
SCALING VIOLATION IN DEEP INELASTIC SCATTERING

structure functions:

\[ F_1(x) \rightarrow F_1(x, Q^2) \]
\[ F_2(x) \rightarrow F_2(x, Q^2) \]

\[ \downarrow \]

\[ \ln Q^2 \text{ dependence} \ (Q^2 = -q^2) \]

\[ \uparrow \]

parton interactions
PDFs and factorization of DIS

- asymptotic freedom
- factorization

\[ \downarrow \]
PDFs and factorization of DIS

- asymptotic freedom
- factorization

\[ F_i(x, Q^2) = \sum_a \int dz \ C_i^a(x/z, Q^2/\mu^2) \ f_a(z, \mu^2) \]

- \( \mu^2 \) ... factorization scale
- \( a \) ... parton type

\( C_i^a(x/z, Q^2/\mu^2) \) ... coefficient functions

\( f_a(z, \mu^2) \) ... parton distribution functions (PDFs)
PDFs and factorization of DIS

- asymptotic freedom
- factorization

\[ F_i(x, Q^2) = \sum_a \int dz \ C_i^a(x/z, Q^2/\mu^2) \ f_a(z, \mu^2) \]

\( \mu^2 \) ... factorization scale
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\( C_i^a(x/z, Q^2/\mu^2) \) ... coefficient functions \( \rightarrow \) pQCD (\( \alpha_s \) exp.)

\( f_a(z, \mu^2) \) ... parton distribution functions (PDFs)
PDFs and factorization of DIS

- asymptotic freedom
- factorization

\[ F_i(x, Q^2) = \sum_a \int dz \ C_i^a(x/z, Q^2/\mu^2) \ f_a(z, \mu^2) \]

- factorization scale
- parton type

\[ C_i^a(x/z, Q^2/\mu^2) \] coefficient functions [\( \rightarrow \) pQCD (\( \alpha_S \) exp.)

\[ f_a(z, \mu^2) \] parton distribution functions (PDFs)

\[ \rightarrow \text{nonpert. input} + \text{DGLAP evolution equation (pQCD)} \]
Parton distribution functions

- Deeply inelastic scattering

\[ \sum_X X \gamma^* q_1 pxp = 2 \gamma^* q_1 pxp \]

- Probability that parton \( q \) has momentum \( xp \)

- No information on spatial distribution of partons
Electromagnetic form factors

- Form factors → charge distribution

\[ \Gamma^\mu(\gamma^* p \rightarrow p) = \gamma^\mu F_1(Q^2) + \frac{\kappa^p}{2M_p} i \sigma^\mu_\nu q_1^\nu F_2(Q^2) \]

\[ q(b_\perp) \sim \int dq_1 e^{iq_1 b_\perp} F_1(t = q_1^2) \]
Electromagnetic form factors

- Form factors $\rightarrow$ charge distribution

\[
\gamma^\ast (\gamma^\ast p \rightarrow p) = \gamma^\mu F_1(Q^2) + \frac{\kappa p}{2M_p} i \sigma^\mu_\nu q_1 F_2(Q^2)
\]

\[
q(b_\perp) \sim \int dq_1 e^{iq_1 \cdot b_\perp} F_1(t = q_1^2)
\]

\[
q(x) \otimes q(b_\perp)
\]
Electromagnetic form factors

- Form factors → charge distribution

\[ \Gamma^{\mu}(\gamma^* p \rightarrow p) = \gamma^\mu F_1(Q^2) + \frac{\kappa_p}{2M_p} i \sigma^\mu_\nu q_1 \sigma^\nu F_2(Q^2) \]

\[ q(b_\perp) \sim \int dq_1 e^{iq_1 \cdot b_\perp} F_1(t = q_1^2) \]
DIS and Compton scattering

- Deeply inelastic scattering $-q_1^2 \equiv Q^2 \rightarrow \infty$, $x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \rightarrow \text{cte.}$

\[
\sum_X \sigma_{tot}(\gamma^* p \rightarrow X) \propto \text{Im} A(\gamma^* p \rightarrow \gamma^* p)
\]

forward Compton scattering
Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller '92, et al. '94]

\[ P = P_1 + P_2 \]
\[ q = (q_1 + q_2)/2 \]
\[ \Delta = P_2 - P_1 \]
Probing the proton with two photons

Deeply virtual Compton scattering (DVCS) [Müller '92, et al. '94]

\[ -q_1^2 = Q^2 \]
\[ q_2^2 = 0 \]

\( P = P_1 + P_2 \)
\( q = (q_1 + q_2)/2 \)
\( \Delta = P_2 - P_1 \)

generalized Bjorken limit:

\[ -q^2 (\text{DVCS}) \approx Q^2/2 \rightarrow \infty \]
\[ \xi = \frac{-q^2}{2P \cdot q} \rightarrow \text{const (as } x_B) \]
\[ \vartheta = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2} \approx \frac{\eta}{\xi} \text{ DVCS} \approx 1 \]
\[ t = (P_2 - P_1)^2 = \Delta^2 \]

\[ \sigma \propto |A(\gamma^* p \rightarrow \gamma p)|^2 \]
Deeply virtual Compton scattering

- Measured in $e p \rightarrow e \gamma p$

- There is a background process
Deeply virtual Compton scattering

- Measured in $ep \rightarrow e\gamma p$

- There is a background process but it can be used to our advantage:

  \[
  \sigma \propto |A_{DVCS}|^2 + |A_{BH}|^2 + A^*_{DVCS}A_{BH} + A_{DVCS}A^*_{BH}
  \]

- Using $A_{BH}$ as a referent “source” enables measurement of the phase of $A_{DVCS}$
Factorization of DVCS \(\rightarrow\) GPDs

→ cross-section can be expressed in terms of (the squares of)
Compton form factors: \(\mathcal{H}(\xi, t, Q^2), \mathcal{E}(\xi, t, Q^2), \tilde{\mathcal{H}}(\xi, t, Q^2), \tilde{\mathcal{E}}(\xi, t, Q^2), \ldots\)

\[\begin{align*}
-q_1^2 &= Q^2 \\
q_2^2 &= 0 \\
\gamma^* &= \gamma \\
x + \xi &\quad 2 \\
x - \xi &\quad 2
\end{align*}\]

Compton form factor is a convolution:

\[a\mathcal{H}(\xi, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/\mu^2) \ H^a(x, \eta = \xi, t, \mu^2)\]

\(H^a(x, \eta, t, \mu^2)\) — Generalized parton distribution (GPD)
Factorization of DVCS $\rightarrow$ GPDs

- $C^a(x, \xi, Q^2/\mu^2)$ ... hard scattering amplitude
  $\rightarrow$ pQCD
Factorization of DVCS $\rightarrow$ GPDs

- $C^a(x, \xi, Q^2/\mu^2)$ ... hard scattering amplitude
  $\rightarrow$ pQCD

- $H^a(x, \eta = \xi, t, \mu^2)$ ... GPD
  $\rightarrow$ nonperturbative input
  $\rightarrow$ evolution $\leftrightarrow$ pQCD (limiting cases DGLAP ($\eta = 0$) and ERBL ($\eta = 1$) evolution equations)

$$\mu^2 \frac{d}{d\mu^2} F(x, \eta, t, \mu^2) = \int_{-1}^{1} \frac{dy}{2\eta} V \left( \frac{\eta + x}{2\eta}, \frac{\eta + y}{2\eta}; \eta \big| \alpha_s(\mu) \right) \cdot F(y, \eta, t, \mu^2)$$
Complementary processes

(double) DVCS
\[ \gamma^* p \rightarrow \gamma^* p \]
\((ep \rightarrow ep l^+ l^-)\)

spacelike DVCS
\[ \gamma^* p \rightarrow \gamma p \]
\((ep \rightarrow ep \gamma)\)

timelike DVCS
\[ \gamma p \rightarrow \gamma^* p \]
\((\gamma p \rightarrow pl^+ l^-)\)

Deeply virtual production of mesons (DVMP)
more difficult, but access to flavours
\[ \gamma^* p \rightarrow Mp \]

factorization: [Collins, Frankfurt, Strikman '97]
Hard-scattering amplitudes (DV processes vs. meson form factors)

**DVCS**

\[ \gamma^* q \rightarrow \gamma q, \quad \gamma^* g \rightarrow \gamma g \]
Hard-scattering amplitudes (DV processes vs. meson form factors)

\[ \gamma^* q \rightarrow \gamma q, \; \gamma^* g \rightarrow \gamma g \]

\[ \gamma^* \gamma \rightarrow (q\bar{q}), \; \gamma^* \gamma \rightarrow (gg) \]

NLO: [Ji et al, Belitsky et al, Mankiewicz et al, '97]
Hard-scattering amplitudes (DV processes vs. meson form factors)

\[ \gamma^* q \rightarrow (q\bar{q})q, \quad \gamma^* g \rightarrow (q\bar{q})g \]

NLO: [Belitsky and Müller '01, Ivanov et al '04]
Hard-scattering amplitudes (DV processes vs. meson form factors)

\[ \gamma^* q \rightarrow (q\bar{q})q, \; \gamma^* g \rightarrow (q\bar{q})g \]

Meson em form factor

\[ \gamma^* (q\bar{q}) \rightarrow (q\bar{q}) \]

NLO: [Belitsky and Müller ’01, Ivanov et al ’04]
Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

\[
\tilde{F}^q(x, \eta, t = \Delta^2) = \int \frac{dz^-}{2\pi} e^{ix^+z^-} \langle P_2 | \bar{q}(-z)\gamma^+\gamma_5q(z) | P_1 \rangle \bigg|_{z^+=0, z_\perp=0}
\]

\[
\tilde{F}^g(x, \eta, t = \Delta^2) = \frac{4}{P^+} \int \frac{dz^-}{2\pi} e^{ix^+z^-} \langle P_2 | G^{+\mu}(-z)\tilde{G}_{a\mu}^+(z) | P_1 \rangle \bigg| ...
\]

- Decomposing into helicity conserving and non-conserving part:

\[
F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2)i\sigma^{+\nu} u(P_1)\Delta_\nu}{2MP^+} E^a \quad a = q, g
\]
Properties of GPDs

- **Forward limit** ($\Delta \to 0, \eta \to 0$): $\Rightarrow \tilde{H}$-GPDs $\to$ PDFs

\[
\begin{align*}
\frac{x + \eta}{2} P^+ & \quad \frac{x - \eta}{2} P^+ \\
\frac{1 + \eta}{2} P^+ & \quad \frac{1 - \eta}{2} P^+
\end{align*}
\]

Forward limit
$\Delta \to 0$

$P = P_1 + P_2$
$\Delta = P_2 - P_1, \Delta^2 = t$
$\eta = -\frac{\Delta^+}{P^+}$
Properties of GPDs

- **Forward limit** \((\Delta \to 0, \eta \to 0)\): \(\tilde{H}\)-GPDs \(\to\) PDFs

\[
\frac{x + \eta}{2} P^+ \quad \frac{x - \eta}{2} P^+ \quad \frac{1 + \eta}{2} P^+ \quad \frac{1 - \eta}{2} P^+
\]

\[\text{GPD} \quad \text{PDF} \]

- **Sum rules**: \(\Rightarrow\) GPD \(\to\) form factors

\[
\sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{l} H_q(x, \eta, t) \\ E_q(x, \eta, t) \end{array} \right\} = \begin{array}{l} F_1(t) \\ F_2(t) \end{array}
\]

\[P = P_1 + P_2 \]

\[\Delta = P_2 - P_1, \quad \Delta^2 = t \]

\[\eta = -\frac{\Delta^+}{P^+} \]
Properties of GPDs

- Forward limit \((\Delta \to 0, \eta \to 0)\): \(\tilde{H}\)-GPDs \(\to\) PDFs

\[
\frac{x + \eta}{2} p^+ + \frac{x - \eta}{2} p^+ \quad \xrightarrow{\text{Forward limit}} \quad x p^+ \quad \xrightarrow{\Delta \to 0} \quad \tilde{H}\text{-GPDs} \to \text{PDFs}
\]

- Sum rules: \(\Rightarrow\) GPD \(\to\) form factors

\[
\sum_{q = u, d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{l}
H^q(x, \eta, t) \\
E^q(x, \eta, t)
\end{array} \right\} = \left\{ \begin{array}{l}
F_1(t) \\
F_2(t)
\end{array} \right\}
\]

- Possibility of solution of proton spin problem

\[
\frac{1}{2} \int_{-1}^{1} dx \frac{1}{x} \left[ H^q(x, \eta, t) + E^q(x, \eta, t) \right] = J^q(t) \quad \text{[Ji '96]}
\]
Properties of GPDs

- Forward limit ($\Delta \to 0, \eta \to 0$): $\Rightarrow \tilde{H}$-GPDs $\to$ PDFs

\[ \frac{x + \eta}{2} P^+ \quad \frac{x - \eta}{2} P^+ \quad \frac{1 + \eta}{2} P^+ \quad \frac{1 - \eta}{2} P^+ \]

- Sum rules: $\Rightarrow$ GPD $\to$ form factors

\[ \sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{l} H^q(x, \eta, t) \\ E^q(x, \eta, t) \end{array} \right\} = \left\{ \begin{array}{l} F_1(t) \\ F_2(t) \end{array} \right\} \]

- Possibility of solution of proton spin problem

\[ \frac{1}{2} \int_{-1}^{1} dx x \left[ H^q(x, \eta, t) + E^q(x, \eta, t) \right] = J^q(t) \quad [\text{Ji '96}] \]

- Polynomiality and positivity constraints
GPDs & PDFs

Deeply Virtual Exclusive Processes & GPDs

Deep Inelastic Scattering & PDFs

[V. D. Burkert, 2006]
Contemporary hierarchy of parton distributions

W.-D. Nowak, Access to GPDs at COMPASS (Primosten/Croatia, Sept. 10-16, 2014)
### Experimental status

#### DVCS

![Graph showing DVCS data](image)

#### DVMP

- in the last decade: vector meson ($\rho, J/\Psi, \phi$) production at H1 and ZEUS, COMPASS, pseudoscalar mesons ($\pi, \eta$) at CLAS . . .

- new results from COMPASS, JLab12 (EIC)

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[from Kumericki et al. 2015]
Towards unravelling GPDs

**DVCS: Compton form factors**

\[
a^a H(\xi, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/\mu^2)) \ H^a(x, \xi, t, \mu^2)
\]

**DVMP: transition form factors**

\[
a^a T(\xi, t, Q^2) = \int dx \int dy \ T^a(x, \xi, y, Q^2/\mu^2)) \ H^a(x, \xi, t, \mu^2) \ \phi(y, \mu^2)
\]

- Complete deconvolution is impossible and to extract GPDs from the experiment we need to model their functional dependence, or alternatively model form factors for start.
Towards unravelling GPDs

DVCS: Compton form factors

\[ a^\mathcal{H}(\xi, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/\mu^2) \ H^a(x, \xi, t, \mu^2) \]

\[ a=\bar{q}, G \text{ or NS,S}(\Sigma, G) \]

DVMP: transition form factors

\[ a^{\mathcal{T}}(\xi, t, Q^2) = \int dx \int dy \ T^a(x, \xi, y, Q^2/\mu^2) \ H^a(x, \xi, t, \mu^2) \phi(y, \mu^2) \]

- Complete deconvolution is impossible and to extract GPDs from the experiment we need to model their functional dependence, or alternatively model form factors for start.

- "Curse of the dimensionality"
  When the dimensionality increases, the volume of the space increases so fast that the available data become sparse.
Towards unravelling GPDs

DVCS: Compton form factors

\[ a^a \mathcal{H}(\xi, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/\mu^2)) \ H^a(x, \xi, t, \mu^2) \]

DVMP: transition form factors

\[ a^a \mathcal{T}(\xi, t, Q^2) = \int dx \ \int dy \ T^a(x, \xi, y, Q^2/\mu^2)) \ H^a(x, \xi, t, \mu^2) \ \phi(y, \mu^2) \]

- Complete deconvolution is impossible and to extract GPDs from the experiment we need to model their functional dependence, or alternatively model form factors for start.
- "Curse of the dimensionality"
  When the dimensionality increases, the volume of the space increases so fast that the available data become sparse.
- Known GPD constraints don’t decrease the dimensionality of the GPD domain space.
Modeling venues

- double distribution amplitude (DDA) satisfy automatically the polinomiality constraint so many models based on it, or specifically Radyushkin’s DDA (RDDA) (VGG code, [Goeke et al. 01], BMK model [Belitsky, Muller, Kirchner 01], GK model [Goloskokov, Kroll 05])

- 'aligned jet' model [Freund, McDermott, Strikman 02], polynomials [Belitsky et al. '98], [Liuti et al. '07], [Moutarde '09]

- 'dual model’ [Polyakov, Shuvaev 02], [Guzey, Teckentrup 06], [Polyakov 07]

- various models in Mellin-Barnes integral representation [Kumericki, Muller, Passek-K 08, ...]

- fitting Compton form factors with neural networks [Kumericki, Muller, Schaefer 11]
Modeling venues

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- 'aligned jet' model [Freund, McDermott, Strikman 02], polynomials [Belitsky et al. '98], [Liuti et al. '07], [Moutarde '09]

- 'dual model' [Polyakov, Shuvaev 02], [Guzey, Teckentrup 06], [Polyakov 07]

- various models in Mellin-Barnes integral representation [Kumericki, Muller, Passek-K 08, ...]

- fitting Compton form factors with neural networks [Kumericki, Muller, Schaefer 11]
DVCS using Mellin-Barnes representation, going to higher-orders and fitting GPDs

K. Kumerički, D. Müller, K. Passek-K.,
*Towards a fitting procedure for deeply virtual Compton scattering at next-to-leading order and beyond*, [hep-ph/0703179]

D. Müller, K. Passek-K., T. Lautenschlager, A. Schäfer,
*Towards a fitting procedure to deeply virtual meson production - the next-to-leading order case*, [arXiv:1310.5394]


T. Lautenschlager, D. Müller and A. Schäfer, [arXiv:1312.5493]
factorization formula for singlet DVCS CFFs:

\[ S \mathcal{H}(\xi, t, Q^2) = \int dx \ C(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ H(x, \xi, t, \mu^2) \]
factorization formula for singlet DVCS CFFs:

\[ S \mathcal{H}(\xi, t, Q^2) = \int dx \ C(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ H(x, \xi, t, \mu^2) \]

\[ = 2 \sum_{j=0}^{\infty} \xi^{-j-1} C_j(Q^2/\mu^2, \alpha_s(\mu)) \ H_j(\xi = \eta, t, \mu^2) \]

\[ H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} dx \ \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots) \]

\[ H_j^a \text{ even polynomials in } \eta \text{ with maximal power } \eta^{j+1} \]
factorization formula for singlet DVCS CFFs:

\[ S \mathcal{H}(\xi, t, Q^2) = \int dx \ C(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ H(x, \xi, t, \mu^2) \]

... in terms of conformal moments

(analogous to Mellin moments in DIS: \( x^n \rightarrow C_n^{3/2}(x), C_n^{5/2}(x) \)):

\[ = 2 \sum_{j=0}^{\infty} \xi^{-j-1} C_j(Q^2/\mu^2, \alpha_s(\mu)) \ H_j(\xi = \eta, t, \mu^2) \]

\[ H_j^a(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} dx \ \eta^{j-1} C_j^{3/2}(x/\eta) H^a(x, \eta, \ldots) \]

\( H_j^a \) even polynomials in \( \eta \) with maximal power \( \eta^{j+1} \)

series summed using Mellin-Barnes integral over complex \( j \):

\[ = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \ \left[ i + \tan \left( \frac{\pi j}{2} \right) \right] \xi^{-j-1} C_j(Q^2/\mu^2, \alpha_s(\mu)) \ H_j(\xi, t, \mu^2) \]
Advantages of conformal moments and Mellin-Barnes representation

- enables simpler inclusion of evolution effects
- powerful analytic methods of complex $j$ plane are available (similar to complex angular momentum of Regge theory)
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Modelling conformal moments

\[ H_j(\eta, t) = \left( \frac{N'_\Sigma F_\Sigma(t)}{N'_G F_G(t)} B(1 + j - \alpha_{\Sigma}(0), 8) \right) + \left( \frac{S_{\Sigma}}{S_G} \right) \text{(subleading partial waves, } \eta^- \text{ dependence!)} \]

- **Leading wave** – simplest case:
  (at NLO data can be fitted with leading wave only)
  - Regge-inspired ansatz
    \[ \alpha_a(t) = \alpha_a(0) + 0.15 t \]
    \[ F_a(t) = \frac{j + 1 - \alpha(0)}{j + 1 - \alpha(t)} \left( 1 - \frac{t}{M_0^a} \right)^{-p_a} \]

- for \( t = 0 \) corresponds to x-space PDFs of the form
  \[ \Sigma(x) = N'_\Sigma x^{-\alpha_{\Sigma}(0)} (1 - x)^7 ; \quad G(x) = N'_G x^{-\alpha_G(0)} (1 - x)^5 \]

- fit parameters: \( N_\Sigma, \alpha_{\Sigma}(0), \alpha_G(0) \) (DIS) and \( M_0^\Sigma \) (DVCS)
  \( (M_0^G = \sqrt{0.7} \text{ GeV from } J/\Psi \text{ prod.}) \)
NLO and NNLO corrections

for generic parameters

\[ \delta^P K = \left| \frac{\mathcal{H}^{NP \text{LO}}}{\mathcal{H}^{NP-1 \text{LO}}} \right| - 1 \], \quad \delta^P \varphi = \text{arg} \left( \frac{\mathcal{H}^{NP \text{LO}}}{\mathcal{H}^{NP-1 \text{LO}}} \right) \]

Thick lines:
"hard" gluon
\( N_G = 0.4 \)
\( \alpha_G(0) = \alpha_S(0) + 0.05 \)

Thin lines:
"soft" gluon
\( N_G = 0.3 \)
\( \alpha_G(0) = \alpha_S(0) - 0.02 \)
Fits (GeParD output)
NNLO fit to HERA DVCS+DIS data

\[ \frac{d\sigma}{dt} (\gamma^* p \rightarrow \gamma p) [\text{nb/GeV}^2] \]

\[ \sigma (\gamma^* p \rightarrow \gamma p) [\text{nb}] \]

\[ F_2 \]

- H1, \( Q^2 = 4 \text{ GeV}^2 \)
- H1, \( Q^2 = 8 \text{ GeV}^2 \)
- ZEUS, \( Q^2 = 9.6 \text{ GeV}^2 \)
- ZEUS, \( W = 82 \text{ GeV} \)
- ZEUS, \( W = 89 \text{ GeV} \)
GPD page and server

- Durham-like CFF/GPD server page

![GPD Server](image)

- Binary code for cross sections and KM models available at http://calculon.phy.hr/gpd/
Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on $x$ and transversal distance $b$

  $H(x, \vec{b}) = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} e^{-i \vec{b} \cdot \vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2)$,

- Average transversal distance:

  $\langle \vec{b}^2 \rangle(x, Q^2) = \frac{\int d\vec{b} \, \vec{b}^2 H(x, \vec{b}, Q^2)}{\int d\vec{b} \, H(x, \vec{b}, Q^2)} = 4B(x, Q^2)$

  (at $Q^2 = 4 \text{ GeV}^2$)

  $\langle \vec{b}^2 \rangle_{\text{gluon}}(\xi = 10^{-3}) = 0.30^{+0.07}_{-0.04} \text{ fm}^2$
Three-dimensional image of a proton

Quarks:

Gluons:
Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVMP... different processes offer different insight and should provide more complete picture.
Summary

- Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVMP ... different processes offer different insight and should provide more complete picture.
- Extraction of GPDs is extremely challenging but efforts for global fits are being made.
- New data are expected from COMPASS and JLab12. DVCS and related processes have a large role in EIC proposal.

The End