Generalized Parton Distributions (GPDs) through DVCS and DVMP

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Outline

1. Introduction
   - Resolving nucleon structure (form factors, PDFs, ...)

2. DVCS, DVMP, GPDs — theory
   - Deeply virtual Compton scattering (DVCS)
   - ... , deeply virtual meson electroproduction (DVMP)
   - Generalized parton distributions (GPDs)

3. DVCS, DVMP, GPDs — phenomenology
   - Experimental status
   - Towards unravelling GPDs
   - Modeling venues
   - One example approach...

4. Summary
Resolving nucleon structure

SCATTERING

→ elastic \( (e^- p \rightarrow e^- p) \)
→ inelastic \( (e^- p \rightarrow e^- \pi p) \)
(\(e^- p \rightarrow e^- X\))

\{ \) exclusive

\{ \) inclusive
Resolving nucleon structure

SCATTERING

→ elastic \((e^- p \rightarrow e^- p)\)  
→ inelastic \((e^- p \rightarrow e^- \pi p)\) \((e^- p \rightarrow e^- X)\)

} exclusive

} inclusive
Parton distribution functions

- Deeply inelastic scattering

\[ -q_1^2 \equiv Q^2 \to \infty, \quad x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \to \text{cte.} \] (Bjorken limit)
Parton distribution functions

- Deeply inelastic scattering

\[-q_1^2 \equiv Q^2 \to \infty, \quad x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \to \text{cte.} \quad \text{(Bjorken limit)}\]
Parton distribution functions

- Deeply inelastic scattering
  
  $-q_1^2 \equiv Q^2 \rightarrow \infty$, $x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \rightarrow \text{cte.}$  
  (Bjorken limit)

- no information on spatial distribution of partons
Electromagnetic form factors

- Form factors $\rightarrow$ charge distribution

$$\Gamma^\mu(\gamma^* p \rightarrow p) = \gamma^\mu F_1(Q^2) + \frac{\kappa p}{2M_p} i \sigma^\mu_\nu q_1^\nu F_2(Q^2)$$

$$q(b_\perp) \sim \int d q_1 \ e^{i q_1 \cdot b_\perp} F_1(t = q_1^2)$$
Electromagnetic form factors

- Form factors → charge distribution

\[ \Gamma^\mu(\gamma^* p \rightarrow p) = \gamma^\mu F_1(Q^2) + \frac{k_p}{2M_p} i \sigma^\mu \gamma_1 F_2(Q^2) \]

\[ q(b_\perp) \sim \int dq_1 \ e^{iq_1 \cdot b_\perp} F_1(t = q_1^2) \]

\[ q(x) \otimes q(b_\perp) \]
Electromagnetic form factors

- Form factors $\rightarrow$ charge distribution

\[
\Gamma^\mu(\gamma^* p \rightarrow p) = \gamma^\mu F_1(Q^2) + \frac{\kappa p}{2M_p} i \sigma^\mu_\nu q_1^\nu F_2(Q^2)
\]

\[
q(b_\perp) \sim \int dq_1 e^{iq_1 \cdot b_\perp} F_1(t = q_1^2)
\]
DIS and Compton scattering

- Deeply inelastic scattering \(-q_1^2 \equiv Q^2 \to \infty\), \(x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \to \text{cte.}\)

\[
\sum_X X = \sum_X \gamma^* p p P D F q(x) = \sigma_{\text{tot}}(\gamma^* p \to X) \propto \text{optical theorem, Im } A(\gamma^* p \to \gamma^* p)
\]

forward Compton scattering
Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller ’92, et al. ’94]

\[
\begin{align*}
-q_1^2 &= Q^2 \\
q_2^2 &= 0
\end{align*}
\]

\[
P &= P_1 + P_2 \\
q &= (q_1 + q_2)/2 \\
\Delta &= P_2 - P_1
\]
Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller ’92, et al. ’94]

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P &= P_1 + P_2 \\
q &= (q_1 + q_2)/2 \\
\Delta &= P_2 - P_1
\]

generalized Bjorken limit:

\[
\xi = -q^2 / 2P \cdot q \rightarrow \text{const (as } x_B) \\
\phi &= \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2} \approx \frac{\eta}{\xi} \overset{\text{DVCS}}{=} 1 \\
t &= (P_2 - P_1)^2 = \Delta^2
\]

\[
\sigma \propto |A(\gamma^* p \rightarrow \gamma p)|^2
\]
Deeply virtual Compton scattering

- Measured in $ep \rightarrow e\gamma p$

- There is a background process but it can be used to our advantage:

$$\sigma \propto |A_{DVCS}|^2 + |A_{BH}|^2 + A_{DVCS}^* A_{BH} + A_{DVCS} A_{BH}^*$$

- Using $A_{BH}$ as a referent “source” enables measurement of the phase of $A_{DVCS}$
Factorization of DVCS $\rightarrow$ GPDs

→ cross-section can be expressed in terms of (the squares of)

Compton form factors: $\mathcal{H}(\xi, t, Q^2)$, $\mathcal{E}(\xi, t, Q^2)$, $\tilde{\mathcal{H}}(\xi, t, Q^2)$, $\tilde{\mathcal{E}}(\xi, t, Q^2)$, ...

[Collins and Freund ’99]

\[ -q_1^2 = Q^2 \quad \gamma^* \quad q_2^2 = 0 \quad \gamma \]

- Compton form factor is a convolution:

\[ a\mathcal{H}(\xi, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/\mu^2) \ H^a(x, \eta = \xi, t, \mu^2) \]

- $H^a(x, \eta, t, \mu^2)$ — Generalized parton distribution (GPD)
Factorization of DVCS $\rightarrow$ GPDs

$$a \mathcal{H}(\xi, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ H^a(x, \eta = \xi, t, \mu^2)$$

- $C^a(x, \xi, Q^2/\mu^2)$ … hard scattering amplitude

$\rightarrow$ pQCD
Factorization of DVCS $\rightarrow$ GPDs

\[ a\mathcal{H}(\xi, t, Q^2) = \int dx \, C^a(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \, H^a(x, \eta = \xi, t, \mu^2) \]

- \( C^a(x, \xi, Q^2/\mu^2) \) ... hard scattering amplitude
  - $\rightarrow$ pQCD

- \( H^a(x, \eta = \xi, t, \mu^2) \) ... GPD
  - $\rightarrow$ nonperturbative input
  - $\rightarrow$ evolution $\leftrightarrow$ pQCD (limiting cases DGLAP ($\eta = 0$) and ERBL ($\eta = 1$) evolution equations)

\[ \mu^2 \frac{d}{d\mu^2} F(x, \eta, t, \mu^2) = \int_{-1}^{1} dy \frac{1}{2\eta} \, \mathbf{V} \left( \frac{\eta + x}{2\eta}, \frac{\eta + y}{2\eta} ; \eta \big| \alpha_s(\mu) \right) \cdot F(y, \eta, t, \mu^2) \]
Complementary processes

(double) DVCS
\[ \gamma^* p \rightarrow \gamma^* p \]  
([ep \rightarrow ep l^+ l^-])

spacelike DVCS
\[ \gamma^* p \rightarrow \gamma p \]  
([ep \rightarrow ep \gamma])

timelike DVCS
\[ \gamma p \rightarrow \gamma^* p \]  
([\gamma p \rightarrow pl^+ l^-])

Deeply virtual production of mesons (DVMP)
more difficult, but access to flavours
\[ \gamma^* p \rightarrow Mp \]

factorization: [Collins, Frankfurt, Strikman ’97]
Hard-scattering amplitudes

DVCS

\[ \gamma^* q \rightarrow \gamma q, \quad \gamma^* g \rightarrow \gamma g \]

NLO: [Ji et al, Belitsky et al, Mankiewicz et al, '97]
Hard-scattering amplitudes

DVMP

\[ \gamma^* q \rightarrow (q\bar{q})q, \quad \gamma^* g \rightarrow (q\bar{q})g \]

NLO: [Belitsky and Müller ’01, Ivanov et al ’04]
Definition of GPDs

- In QCD GPDs are defined as [Müller ’92, et al. ’94, Ji, Radyushkin ’96]

\[
\tilde{F}^q(x, \eta, t = \Delta^2) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+\gamma_5 q(z)|P_1 \rangle \Big|_{z^+ = 0, z_\perp = 0}
\]

\[
\tilde{F}^g(x, \eta, t = \Delta^2) = \frac{4}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2|G_{a^+}^+(-z)\tilde{G}_{a^+}^+(z)|P_1 \rangle \Big|_{t = 0}
\]

- Decomposing into helicity conserving and non-conserving part:

\[
F^a = \frac{\bar{u}(P_2)\gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2)i\sigma^{+\nu} u(P_1)\Delta_{\nu}}{2MP^+} E^a \quad a = q, g
\]
Properties of GPDs

- **Forward limit** \((\Delta \to 0, \eta \to 0)\): \( \Rightarrow \tilde{H}\text{-GPDs} \to \text{PDFs} \)

\[
\begin{align*}
\frac{x + \eta}{2} p^+ & \quad \frac{x - \eta}{2} p^+ \\
1 + \eta p^+ & \quad 1 - \eta p^+ \\
\end{align*}
\]

\[
P = P_1 + P_2 \\
\Delta = P_2 - P_1, \quad \Delta^2 = t \\
\eta = -\frac{\Delta^+}{p^+}
\]
Properties of GPDs

- **Forward limit** \((\Delta \to 0, \eta \to 0)\): \(\Rightarrow \tilde{H}\)-GPDs \(\to\) PDFs

\[
\begin{align*}
\frac{x + \eta}{2} p^+ & \quad \frac{x - \eta}{2} p^+ \\
1 + \eta p^+ & \quad 1 - \eta p^+
\end{align*}
\rightarrow
\begin{align*}
x p^+ & \quad x p^+ \\
p^+ & \quad p^+
\end{align*}
\]

\[P = P_1 + P_2, \quad \Delta = P_2 - P_1, \quad \Delta^2 = t, \quad \eta = -\frac{\Delta^+}{P^+}\]

- **Sum rules:** \(\Rightarrow\) GPD \(\to\) form factors

\[
\sum_{q=u,d} Q_q \int_{-1}^{1} dx \begin{bmatrix}
H^q(x, \eta, t) \\
E^q(x, \eta, t)
\end{bmatrix} = \begin{bmatrix}
F_1(t) \\
F_2(t)
\end{bmatrix}
\]
Properties of GPDs

- **Forward limit** ($\Delta \to 0, \eta \to 0$): $\Rightarrow \tilde{H}$-GPDs $\to$ PDFs

\[
\begin{align*}
\frac{x + \eta}{2} P^+ & \to \tilde{H}_u
\
\frac{x - \eta}{2} P^+ & \to \tilde{H}_d
\
\frac{1 + \eta}{2} P^+ & \to H_u
\
\frac{1 - \eta}{2} P^+ & \to H_d
\end{align*}
\]

- **Sum rules:** $\Rightarrow$ GPD $\to$ form factors

\[
\sum_{q=u,d} Q_q \int_{-1}^{1} dx \begin{cases} H^q(x, \eta, t) \\ E^q(x, \eta, t) \end{cases} = \begin{cases} F_1(t) \\ F_2(t) \end{cases}
\]

- **Possibility of solution of proton spin problem**

\[
\frac{1}{2} \int_{-1}^{1} dx x \left[ H^q(x, \eta, t) + E^q(x, \eta, t) \right] = J^q(t) \quad \text{[Ji '96]}
\]
Properties of GPDs

- **Forward limit** ($\Delta \to 0$, $\eta \to 0$): $\Rightarrow \tilde{H}$-GPDs $\to$ PDFs

  \[ \frac{x + \eta}{2} p^+ + \frac{x - \eta}{2} p^+ \quad \overset{\text{Forward limit}}{\xrightarrow{\Delta \to 0}} \quad \frac{1 + \eta}{2} p^+ + \frac{1 - \eta}{2} p^+ \]

  \[ P = P_1 + P_2 \]
  \[ \Delta = P_2 - P_1, \quad \Delta^2 = t \]
  \[ \eta = -\frac{\Delta}{P^+} \]

- **Sum rules:** $\Rightarrow$ GPD $\to$ form factors

  \[ \sum_{q=u,d} Q_q \int_{-1}^{1} dx \left\{ \begin{array}{l} H^q(x, \eta, t) \\ E^q(x, \eta, t) \end{array} \right\} = \begin{array}{l} F_1(t) \\ F_2(t) \end{array} \]

- **Possibility of solution of proton spin problem**

  \[ \frac{1}{2} \int_{-1}^{1} dx x \left[ H^q(x, \eta, t) + E^q(x, \eta, t) \right] = J^q(t) \]  
  
  [Ji ’96]

- **Polynomiality and positivity constraints**
Contemporary hierarchy of parton distributions

- **GTMD**
  \[ X(x, \xi, \Delta , k_T, k_T, t) \]
  - Take limit $\xi = \Delta = 0$
  - Integrate over $k_T$

- **TMD**
  \[ h_1(x, k_T) \]
  - Integrate over $k_T$

- **GPD**
  \[ H(x, \xi, t) \]
  - Take limit $\xi = t = 0$
  - Integrate over $k_T$

- **PDF**
  \[ q(x) \]
  - \[ F(t) \]

- **WF**
  \[ X(x, \xi, b_\perp, k_T, k_T, b_\perp) \]
  - \[ \xi = 0 \]
  - Integrate over $k_T$

- **WD**
  - \[ \xi = 0 \]
  - Integrate over $x$

- **Spin Densities**

- **Charge Densities**

*Courtesy M. Murray, Glasgow*

W. D. Nowak, Access to GPDs at COMPASS (Princetown/Croatia, Sept. 10-16, 2014)
Experimental status

**DVCS**

\[ \text{[from Kumericki et al. 2015]} \]

**DVMP**

- in the last decade: vector meson ($\rho, J/\Psi, \phi$) production at H1 and ZEUS, COMPASS, pseudoscalar mesons ($\pi, \eta$) at CLAS . . .

→ new results from COMPASS, JLab12 (EIC)
Towards unravelling GPDs

**DVCS: Compton form factors**

\[ a^\mathcal{H}(\xi, t, Q^2) = \int dx \ C^a(x, \xi, Q^2/\mu^2)) \ H^a(x, \xi, t, \mu^2) \]

**DVMP: transition form factors**

\[ a^\mathcal{T}(\xi, t, Q^2) = \int dx \ \int dy \ T^a(x, \xi, y, Q^2/\mu^2)) \ H^a(x, \xi, t, \mu^2) \phi(y, \mu^2) \]

- **Complete deconvolution is impossible** and to extract GPDs from the experiment we need to **model** their functional dependence, or alternatively model form factors for start.
- **”Curse of the dimensionality”**
  When the dimensionality increases, the volume of the space increases so fast that the **available data become sparse**.
- Known GPD constraints don’t decrease the dimensionality of the GPD domain space.
double distribution amplitude (DDA) satisfy automatically the polynomiality constraint so many models based on it, or specifically Radyushkin's DDA (RDDA) (VGG code, [Goeke et al. 01], BMK model [Belitsky, Muller, Kirchner 01], GK model [Goloskokov, Kroll 05]))

'aligned jet' model [Freund, McDermott, Strikman 02], polynomials [Belitsky et al. ’98], [Liuti et al. ’07], [Moutarde ’09]

'dual model' [Polyakov, Shuvaev 02], [Guzey, Teckentrup 06], [Polyakov 07]

various models in Mellin-Barnes integral representation [Kumericki, Muller, Passek-K 08, ...]

fitting Compton form factors with neural networks [Kumericki, Muller, Schaefer 11]
factorization formula for singlet DVCS CFFs:

\[ S \mathcal{H}(\xi, t, Q^2) = \int dx \ C(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ H(x, \xi, t, \mu^2) \]
factorization formula for singlet DVCS CFFs:

\[ S \mathcal{H}(\xi, t, Q^2) = \int dx \ C(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ H(x, \xi, t, \mu^2) \]

... in terms of conformal moments

(Analogous to Mellin moments in DIS: \( x^n \rightarrow C_n^{3/2}(x), C_n^{5/2}(x) \)):

\[ = 2 \sum_{j=0}^{\infty} \xi^{-j-1} C_j(Q^2/\mu^2, \alpha_s(\mu)) \ H_j(\xi = \eta, t, \mu^2) \]

\[ H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} dx \ \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \ldots) \]

... \( H_j^a \) even polynomials in \( \eta \) with maximal power \( \eta^{j+1} \)
factorization formula for singlet DVCS CFFs:

\[
S\mathcal{H}(\xi, t, Q^2) = \int dx \ C(x, \xi, Q^2/\mu^2, \alpha_s(\mu)) \ \mathcal{H}(x, \xi, t, \mu^2)
\]

... in terms of conformal moments

(analogous to Mellin moments in DIS: \(x^n \to C_n^{3/2}(x), C_n^{5/2}(x)\)):

\[
= 2 \sum_{j=0}^{\infty} \xi^{-j-1} C_j(Q^2/\mu^2, \alpha_s(\mu)) \ \mathcal{H}_j(\xi = \eta, t, \mu^2)
\]

\[
H_j^q(\eta, \ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} dx \ \eta^{j-1} C_j^{3/2}(x/\eta) \mathcal{H}^q(x, \eta, \ldots)
\]

\(H_j^a\) even polynomials in \(\eta\) with maximal power \(\eta^{j+1}\)

series summed using Mellin-Barnes integral over complex \(j\):

\[
= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \left[ i + \tan \left( \frac{\pi j}{2} \right) \right] \xi^{-j-1} C_j(Q^2/\mu^2, \alpha_s(\mu)) \ \mathcal{H}_j(\xi, t, \mu^2)
\]
Advantages of conformal moments and Mellin-Barnes representation

- enables simpler inclusion of evolution effects
- powerful analytic methods of complex $j$ plane are available (similar to complex angular momentum of Regge theory)
- opens the door for interesting modelling of GPDs
- possible efficient and stable numerical treatment $\Rightarrow$ stable and fast computer code for evolution and fitting
- moments are equal to matrix elements of local operators and are thus directly accessible on the lattice

NNLO corrections for DVCS accessible by making use of conformal OPE and known NNLO DIS results
NLO and NNLO corrections

for generic parameters

\[ \delta^P K = \frac{|\mathcal{H}^{NP \text{LO}}|}{|\mathcal{H}^{NP-1 \text{LO}}|} - 1, \]

\[ \delta^P \varphi = \arg \left( \frac{\mathcal{H}^{NP \text{LO}}}{\mathcal{H}^{NP-1 \text{LO}}} \right). \]
Fits (GeParD output)
NNLO fit to HERA DVCS+DIS data

\[ d\sigma(\gamma^* p \rightarrow \gamma p)/dt \ [\text{nb}/\text{GeV}^2] \]

\[ \sigma(\gamma^* p \rightarrow \gamma p) \ [\text{nb}] \]

\[ \sigma(\gamma^* p \rightarrow \gamma p) \ [\text{nb}] \]

\[ F_2 \]

- [H1, \( Q^2 = 4 \text{ GeV}^2 \)]
- [H1, \( Q^2 = 8 \text{ GeV}^2 \)]
- [ZEUS, \( W = 82 \text{ GeV} \)]
- [ZEUS, \( W = 89 \text{ GeV} \)]

\[ \text{H1, } Q^2 = 4 \text{ GeV}^2 \]
\[ \text{H1, } Q^2 = 8 \text{ GeV}^2 \]
\[ \text{ZEUS, } W = 82 \text{ GeV} \]
\[ \text{ZEUS, } W = 89 \text{ GeV} \]
Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on $x$ and transversal distance $b$

  \[ H(x, \vec{b}) = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} e^{-i\vec{b} \cdot \vec{\Delta}} H(x, \eta = 0, \Delta^2 = -\vec{\Delta}^2) , \]

[Burkardt '00, '02]
Three-dimensional image of a proton

Quarks:

Gluons:
Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVMP... different processes offer different insight and should provide more complete picture.
Summary

- Generalized parton distributions offer unified description of the proton structure. They are experimentally accessible via DVCS, DVMP... different processes offer different insight and should provide more complete picture.
- Extraction of GPDs is extremely challenging but efforts for global fits are being made.
- New data are expected from COMPASS and JLab12. DVCS and related processes have a large role in EIC proposal.

The End