Supercapacitors, cell balancing using resistors

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Abstract— Supercapacitors, also known as ultra-capacitors and electrochemical capacitors a special kind of capacitor with the capacity of the order of the farad (tens, hundreds and thousands of farads). The paper describes their basic features and presents their equivalent schemes with the physical meaning of the circuit elements in them. This paper gives an overview of the problem of unequal voltage distribution in circuit with serial connected supercapacitors. Passive voltage balancing techniques are shown. Analytical expressions that describe the behaviour of supercapacitor serial connection with and without the passive resistor balancing circuit is derived. On a practical example are compared the results obtained by derived analytical expressions and simulation.

Keywords— electrochemical double layer capacitors, ultracapacitors, voltage balancing

1. INTRODUCTION

Supercapacitors (SCs), electrochemical double layer capacitors (EDLCs) or ultracapacitors (UCs) are names for energy storage devices which store energy via a physical process of charge separation at the solid/liquid interface [1]. As energy storage devices they have certain advantages and disadvantages compared to capacitors and batteries. They have the ability to store and supply energy with high specific power. This makes them particularly attractive components for certain purposes. Because of the electrostatic nature of the energy storage in them, their durability, i.e. the number of charge and discharge cycles, is relatively high compared to any type of battery, they do not need to be maintained and are operating in a very wide range of temperatures (from -40 °C to + 60 °C to [1-3]). Because of the physical process on which energy storage is based, they regularly come with a relatively low working voltage. Their operating voltage ranges from 2.5 V to 2.7 V (depending on used electrolyte). Since they are very rare applications where such low operating voltage can be directly used, they need to be connected in series to achieve a satisfactory working voltage. As the supercapacitors parameter tolerance is usually 20 %, the voltage distribution on SCs in such a serial connection will not be uniform. In this case, the voltage at individual capacitors may exceed the rated voltage. Usually, the SCs are constructed so that their voltage must not be increased to an amount greater than 0.1 V from the rated voltage. If this happens, electrolysis of electrolytes occurs. If this does not happen, and the working voltage is still very high and close to the rated voltage this will significantly reduce supercapacitor lifetime. To prevent this, various active and passive electronic circuits are designed to achieve a uniform voltage distribution on SCs.

This paper describes the reason for the unequal voltage distribution across SCs in the serial connection. Simple passive balancing techniques have been shown. Particular attention is given to passive voltage balancing using a resistor. Analytical expressions were derived for describing the behaviour of this passive circuit for voltage balancing. In a practical example, the results obtained by derived analytical expressions and simulations were compared.

2. ABOUT SUPERCAPACITORS

Compared to rechargeable batteries, SCs have a high number of charge and discharge cycles (100.000 compared to 1000) and high power per unit of mass (or volume) but can store a small amount of energy per unit of mass (or volume) (Figure 1 [1]).

Figure 1.: Sketch of Ragone plot for various energy storage and conversion devices. The indicated areas are rough guide lines.
Every SC has two electrodes, mechanically separated by a separator, which are ionically connected to each other via the electrolyte. The electrolyte is a mixture of positive and negative ions dissolved in a solvent such as water or organic substance. The electrodes are in a suitable electrolyte and separated by a thin separator (Fig. 2. [4]).

![Activated carbon](image1.png)

Figure 2.: a) Simplified two-dimensional representation of the supercapacitor structure, b) actual construction (cylindrical type)

The other most commonly used SC forms are shown in Figure 3. [4]. The most famous manufacturers are: Samwha, Samxon, Maxwell, Panasonic, Korchip, VINATech, Nesscap [5-11].

![WIMA supercapacitors](image2.png)

Figure 3. WIMA supercapacitors a) with rectangular casing, b) supercapacitor modules

### 3. THE SC PROPERTIES AND ITS MODELS

Depending on the application, SCs can be modeled from the simplest schemes as shown in Figure 4. in which network (circuit) elements have meanings: $R_{ESR}$ - equivalent series resistance, $L_{ESL}$ - equivalent series inductance, $R_0$ - equivalent parallel (leakage resistance $R_L$), $C_0$ - capacitance at zero voltage.

![Equivalent circuit with cascaded RC elements](image3.png)

Figure 4.: Equivalent circuit with cascaded RC elements

All the way up to very complex schemes in which the properties of the electron absorption were taken into account and interdependence of capacity and voltage (Figure 5, [12]).

![General voltage dependence of the supercapacitor capacitance](image4.png)

Figure 5.: General voltage dependence of the supercapacitor capacitance

Supercapacitor differential capacitance depend on the applied voltage. It is usually approximated as a function of voltage:

$$C(u) = C_0 + K_U \cdot u$$

(1)

Where $C_0$ is the initial linear capacitance (at zero voltage) and $K_U$ is a coefficient that takes into account the capacitance variation with respect to applied voltage. Voltage dependence of the SC capacitance can be taken into account by scheme shown on Fig. 6. where $R_0$ denotes equivalent series resistance, $R_1$ denotes internal resistance). One of the schemas by which can be taken into account dielectric absorption is shown in the Fig. 7. where $R_1, R_2, ..., R_n$ denotes distributed internal resistance, and $C_1, C_2, ..., C_n$ denoted distributed capacitance.

![Voltage dependent supercapacitors equivalent scheme](image5.png)

Figure 6.: Voltage dependent supercapacitors equivalent scheme

![Equivalent circuit with cascaded RC elements](image6.png)

Figure 7.: Equivalent circuit with cascaded RC elements
The third particularly pronounced phenomena related to supercapacitors compared to ordinary capacitors is frequency dependence of the supercapacitor capacitance (Figure 8. [13]).

![Figure 8: General frequency dependence of the supercapacitor capacitance](image)

This extraordinary strong frequency dependence can be explained by the different distances the ions have to move in the electrode’s pores. These three phenomena strongly distinguish SCs compared to ordinary capacitors.

4. SUPERCAPACITOR LIFETIME

Lifetime performance of SC is dependent upon the conditions under which the cells or the modules are being used. In general SCs do not have a hard end of life failure similar to batteries. Their end of life is defined as when the capacitance and/or ESR (equivalent series resistance) has degraded beyond the certain limits, usually 80% of the rated value. The lifetime of the SC is most affected by the applied voltage and operating temperature (Figure 9. [14]). Expected life time at room temperature is between 5 and 10 years. The estimated life duration corresponds to the time between the factory measurement and the 80 % decrease of capacitance [7].

![Figure 9: Life expectancy of a SC cell under different temperatures and operating voltages](image)

Simply stated, lifetime is reduced by half when SC temperature increases by 10 °C or when applied voltage is increases by 0.2 V [14, 15]. Relation between the lifetime and operating voltage and temperature can be described by the simple equation [15]:

$$T_{exp}(U, \vartheta) = T_0 \cdot e^{\left(\frac{U}{U_0} - \frac{\vartheta}{D_0}\right)}.$$  \hspace{1cm} (2)

Where: $T_0$ is the static lifetime (s), $U$ is the constant voltage across the supercapacitor terminals (V), and $\vartheta$ hypothetical constant working temperature of the supercapacitor (°C). The numerical values of constants $T_0$, $U_0$ and $D_0$ are [15]: $T_0 = 1.4 \cdot 10^{13}$ (s), $U_0 = 0.2 / \ln 2$ (V), $D_0 = 10 / \ln 2$ (°C).

5. PASSIVE VOLTAGE BALANCING

Due to the low operating voltage of the SCs, except in rare cases where such voltage can be directly used, it is necessary to connect them in series to achieve a suitable working voltage. As the SCs parameter tolerance is usually 20 %, the voltage distribution on SCs in such a serial connection will not be uniform. In this case, the voltage at individual SC may exceed the rated voltage or higher. To prevent this and to equalize the voltage, various techniques are used to balance the voltage across serially connected SCs. These techniques can be divided into two groups, active and passive. Active techniques have a number of advantages over passive techniques, but require additional electronics. In contrast, passive techniques are very simple and despite their shortcomings, they are still very widespread (Fig. 10 [14]).

![Figure 10: Cell balancing circuits a) resistor, b) switched-resistor, c) Zener diodes](image)

6. CELL BALANCING USING RESISTOR

The influence of SC leakage resistance on voltage distribution on SCs during and after the transient can be understood from the equivalent scheme shown in Figure 11.
Although the scheme has only two supercapacitors, without loss of generality by deduction the conclusions can be applied to a series of \( n \) supercapacitors. From the scheme shown in Figure 11, follows:

\[
E - i \cdot R_s - u_{c1} - u_{c2} = 0, \tag{3}
\]

\[
i = i_{RL1} + i_{C1}, \tag{4}
\]

\[
i = i_{RL2} + i_{C2}, \tag{5}
\]

\[
i_{RL1} = \frac{u_{c1}}{R_{L1}}, \tag{6}
\]

\[
i_{RL2} = \frac{u_{c2}}{R_{L2}}, \tag{7}
\]

\[
i_{C1} = C_1 \frac{du_{c1}}{dt}, \tag{8}
\]

\[
i_{C2} = C_2 \frac{du_{c2}}{dt}. \tag{9}
\]

The equations (3-9) describe the behavior of the system formed by two supercapacitors during and after the transient (\( t \geq 0 \)). Inserting equations (4), (6) and (8) into (3) gives

\[
E - \left( \frac{u_{c1}}{R_{L1}} + C_1 \frac{du_{c1}}{dt} \right) R_s - u_{c1} - u_{c2} = 0. \tag{10}
\]

Equations (4) and (5) give equality

\[
i_{RL1} + i_{c1} = i_{RL2} + i_{c2}. \tag{11}
\]

Substituting (6-9) in expression above gives

\[
\frac{u_{c1}}{R_{L1}} + C_1 \frac{du_{c1}}{dt} = \frac{u_{c2}}{R_{L2}} + C_2 \frac{du_{c2}}{dt}. \tag{12}
\]

Taking Laplace transforms [16] of (10) and (12) with initial conditions equal to zero gives:

\[
\frac{E}{s} - \left( \frac{u_{c1}(s)}{R_{L1}} + sC_1 u_{c1}(s) \right) R_s - u_{c1}(s) - u_{c2}(s) = 0, \tag{13}
\]

\[
\frac{u_{c1}(s)}{R_{L1}} + sC_1 u_{c1}(s) = \frac{u_{c2}(s)}{R_{L2}} + sC_2 u_{c2}(s). \tag{14}
\]

Rearranging the above expression shown in three steps gives:

\[
U_{c1}(s) \left( \frac{1}{R_{L1}} + sC_1 \right) = U_{c2}(s) \left( \frac{1}{R_{L2}} + sC_2 \right), \tag{15}
\]

\[
U_{c1}(s) \left( 1 + \frac{sR_{L1}C_1}{R_{L1}} \right) = U_{c2}(s) \left( 1 + \frac{sR_{L2}C_2}{R_{L2}} \right), \tag{16}
\]

\[
U_{c2}(s) = U_{c1}(s) \cdot \frac{R_{L2}}{R_{L1}} \cdot \frac{1 + sR_{L2}C_1}{1 + sR_{L2}C_2}. \tag{17}
\]

Considering that the supercapacitor with higher capacitance always has lower leakage resistance and vice versa, it can be adopted \( R_{L1}C_1 \approx R_{L2}C_2 \).

Accordingly, the expression (17) can be simplified:

\[
U_{c2} \approx U_{c1} \cdot \frac{R_{L2}}{R_{L1}}. \tag{18}
\]

Substituting (18) into (13) gives

\[
U_{c1}(s) \left( \frac{R_s}{R_{L1}} + sR_sC_1 \right) + U_{c1}(s) \left( 1 + \frac{R_{L2}}{R_{L1}} \right) = \frac{E}{s}, \tag{19}
\]

Since the internal resistance of the source \( R_s \) (including the connecting wires) is the order of magnitude mOhms, and the supercapacitor leakage resistance is the order of magnitude kOhms, almost without any significant error is valid the following statement \( R_s/R_{L1} \ll 1 \). Considering that, equation (19) can be simplified as follows:

\[
U_{c1}(s) \cdot sR_sC_1 + U_{c1}(s) \left( 1 + \frac{R_{L2}}{R_{L1}} \right) = \frac{E}{s}. \tag{20}
\]

Rearranging the above expression shown in several steps gives:

\[
U_{c1}(s) \left( \frac{R_{L1} + R_{L2}}{R_{L1}} + sR_sC_1 \right) = \frac{E}{s}, \tag{21}
\]

\[
U_{c1}(s) \frac{R_{L1} + R_{L2}}{R_{L1}} \left( 1 + s \frac{R_s}{R_{L1} + R_{L2}} C_1 \right) = \frac{E}{s}, \tag{22}
\]
Additional arrangement gives the expression for the voltage on the supercapacitor $C_1$:

$$U_{C_1}(s) = \frac{E}{s} \frac{R_{L_1}}{R_{L_1} + R_{L_2}} \cdot \frac{1}{1 + s \frac{R_{S} \cdot R_{L_1}}{R_{L_1} + R_{L_2}} C_1}. \quad (23)$$

By analogous procedure follows:

$$U_{C_2}(s) = \frac{E}{s} \frac{R_{L_2}}{R_{L_1} + R_{L_2}} \cdot \frac{1}{1 + s \frac{R_{S} \cdot R_{L_1}}{R_{L_1} + R_{L_2}} C_2}. \quad (24)$$

Equations (24) and (25) can be concisely written as:

$$U_{C_1}(s) = \frac{E}{s} \frac{R_{L_1}}{R_{L_1} + R_{L_2}} \cdot \frac{1}{1 + \frac{1}{T_1}}, \quad (28)$$

$$U_{C_2}(s) = \frac{E}{s} \frac{R_{L_2}}{R_{L_1} + R_{L_2}} \cdot \frac{1}{1 + \frac{1}{T_2}}. \quad (29)$$

The inverse Laplace Transform of (28) and (29) is carried out according to the expression [16]:

$$L^{-1} \left( \frac{1}{s(s+a)} \right) = \frac{1}{a} \left( 1 - e^{-\frac{t}{T}} \right). \quad (30)$$

Supercapacitors voltages in the time domain are:

$$u_{C_1}(t) = E \cdot \frac{R_{L_1}}{R_{L_1} + R_{L_2}} \left( 1 - e^{-\frac{t}{T_1}} \right), \quad (31)$$

$$u_{C_2}(t) = E \cdot \frac{R_{L_2}}{R_{L_1} + R_{L_2}} \left( 1 - e^{-\frac{t}{T_2}} \right). \quad (32)$$

Accordingly, after the end of transient, supercapacitor which has a higher leakage resistance will take greater voltage and vice versa. Expressions (31) and (32) allow graphical representation of the voltage transition phenomenon on supercapacitors in the way shown in Figure 12.

![Figure 12: Transient voltage change on SCs when no voltage balancing was used](image)

The dynamics of changing the voltage on supercapacitors is determined by time constants $T_1$ and $T_2$, which are defined by expressions (26) and (27). Generally, the time constants $T_1$ and $T_2$ are different, but considering that the supercapacitor with higher capacitance always has lower leakage resistance and vice versa, it can be adopted $R_{L_1}C_1 \approx R_{L_2}C_2$, they can be considered to be almost identical.

7. SIMULATION

Since analytical expressions are obtained by different approximations, their validity needs to be checked by simulation. For this purpose, the ATP-EMTP software [17] was used. The first simulation was performed for the serial connection of two SCs without balancing resistors according to the model in Fig. 11. A ATP-EMTP model of serial connection of two SCs without balancing resistors is shown in Figure 13. The second simulation was performed for the serial connection of two supercapacitors with balancing resistors $R_B$. 

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A ATP-EMTP model of serial connection of two supercapacitors with balancing resistors is shown in Figure 14. Both simulations were conducted for a supercapacitor with nominal capacity of 100 F from the manufacturer SAMWHA [6].

According to the manufacturer [6], the referred SC is designated as: DB, Snap-in Terminal Type, Standard Series, Trade mark: “Green-Cap”. Basic SC data are: Rated voltage $U = 2.7$ V, Capacitance: $C = 100$ F ($\pm/ - 20\%$), Equivalent Series resistance at DC condition $ESR \omega = 10$ mΩ. Equivalent Series Resistance at $1$ kHz, $ESR_{kHz} = 8$ mΩ, Leakage Current $I_L = 0.27$ mA (at 2.7 V).

The simulation parameters are: $E = 5$ V, $R_S = 0.1$ Ω, $R_{L1} = 12$ kΩ, $R_{L2} = 8$ kΩ, $C_1 = 80$ F, $C_2 = 120$ F, $R_B = 1.0$ kΩ.

After performing the simulation wave forms of voltages $u_{c1}$ and $u_{c2}$ were obtained and are shown in Figures 15 and 16.

8. RESULTS AND DISCUSSION

Referring to Figure 15, the voltages $u_{c1}$ and $u_{c2}$ in the steady state ($t \to \infty$) are $u_{c1}(\infty) = 3.0$ V and $u_{c2}(\infty) = 2.0$ V.

According to the analytical expressions, voltages $u_{c1}$ and $u_{c2}$ in new steady state after ending of transient ($t \to \infty$) are determined by the expressions (33) and (34):

$$u_{c1}(\infty) = E \frac{R_{L1}}{R_{L1} + R_{L2}} = 5 \cdot \frac{12 \cdot 10^3}{12 \cdot 10^3 + 8 \cdot 10^3} = 0.60 \cdot 3.0 \text{ V}$$

Thus, the results obtained by analytical expressions and simulation are perfectly matched.

Referring to Figure 16, the voltages $u_{c1}$ and $u_{c2}$ in the steady state ($t \to \infty$) are $u_{c1}(\infty) = 2.6$ V and $u_{c2}(\infty) = 2.40$ V.
When balancing \((R_B = 1.0 \, \text{k} \Omega)\) is used, according to the analytical expressions, voltages \(u_{C1}\) and \(u_{C2}\) in new steady state after ending of transient \((t \rightarrow \infty)\) are:

\[
u_{C1}(x) = E \cdot \frac{R_{L1}}{R_{L1} + R_{L2}} = \frac{5 \cdot 0.92 \cdot 10^{-3}}{0.92 \cdot 10^{-3} + 0.89 \cdot 10^{-3}} = 0.51 \, \text{V}
\]

\[
u_{C2}(x) = E \cdot \frac{R_{L2}}{R_{L1} + R_{L2}} = \frac{5 \cdot 0.89 \cdot 10^{-3}}{0.92 \cdot 10^{-3} + 0.89 \cdot 10^{-3}} = 0.49 \, \text{V}
\]

Where \(R_{L1}\) and \(R_{L2}\) is equivalent resistance given by:

\[
R_{L1} = \frac{R_B \cdot R_{L1}}{R_B + R_{L1}}, \quad (39)
\]

\[
R_{L2} = \frac{R_B \cdot R_{L2}}{R_B + R_{L2}}. \quad (40)
\]

Thus, the results obtained by analytical expressions and simulation although do not match completely, they are very close.

Comparing Figures 15 and 16, it is noted that the addition of voltage balancing resistances slightly affect the time constants. This can also be explained by analytical expressions. For example, the time constant \(T_1\) without balancing, and after balancing \(T_i\) is determined by expressions

\[
T_1 = \frac{R_B \cdot R_{L1}}{R_{L1} + R_{L2}} \cdot C_1, \quad (41)
\]

\[
T_i = \frac{R_B \cdot R_{L1}}{R_{L1} + R_{L2}} \cdot C_1. \quad (42)
\]

By comparing these two terms it follows that change will occur if the following members change the amount:

\[
\frac{R_{L1}}{R_{L1} + R_{L2}}, \quad (43)
\]

\[
\frac{R_{L1}'}{R_{L1} + R_{L2}'}. \quad (44)
\]

Since these members are approximately the same before and after balancing, it turns out that balancing using resistors has a minor impact on time constants. Thus, the results obtained by analytical expressions and simulation for time constants are in good agreement.

9. CONCLUSION

Generally, the use of balancing circuits in supercapacitors serial connection is mandatory. The only thing of choice is which balancing circuit should be used. Balancing voltage on supercapacitors prevents electrolyte electrolysis and prolongs the lifetime of the supercapacitor. If balancing with the resistors is selected for that purpose, then this paper can help in understanding the properties of that technique.

The results of the simulation indicate that the derived analytical expressions faithfully describe the behaviour of the serial connection of the two supercapacitors during the transient and in the steady state conditions. Analytical expressions, although obtained with several approximations, they proved to be extremely accurate and as such, are a useful tool for analyzing the operation of the serial connection of the supercapacitors.

Although passive balancing with resistors improves voltage distribution on supercapacitors, because extra currents passing thru balancing resistors this reduces the energy efficiency of supercapacitors in application as energy storage devices. Therefore, this technique is not recommended for applications where energy efficiency is important. For such applications it is recommended to use circuits for active voltage balancing on supercapacitors.

REFERENCES:


