On approximation of finite-energy sequences of Müller’s functional with non-standard 2-well potential

Andrija Raguž1,∗

1 Zagreb School of Economics and Management, Department of Mathematics and Statistics, Jordanovac 110, 10 000 Zagreb, Croatia

We present the basic results and conjectures regarding possibility of approximating finite-energy sequences of Müller’s functional (which was for the first time, and in its simplest form, studied in paper S. Müller: Singular perturbations as a selection criterion for periodic minimizing sequences, Calc. Var. Partial Differential Equations 1(2), 169–204 (1993)) by 1-Lipschitz and 1-periodic finite-energy sequences. Our results extend known results in the case of simplest pinning term concerning the actual minimizers as small parameter epsilon tends to zero, whereby standard assumption on growth of 2-well potential at infinity (which immediately yields equi-coercivity) is replaced by non-standard one.

1 Formulation of the problem

We consider asymptotic behavior as ε → 0 of finite-energy sequences (FE sequences for short) of (rescaled) Müller’s functional $I_0^ε(v) := 2∫_0^1 (ε^2 v''^2(s) + W(v'(s)) + a(s, v(s), v'(s))v''^2(s))ds$ (cf. [1]), where $v ∈ H^2(0, 1)$, $W$ is 2-well potential, $W ≥ 0$, $W(ξ) = 0$ iff $ξ ∈ {−1, 1}$, $a ∈ L^1_{per}(0, 1); C_0(ℝ^2))$ is Carathéodory function such that, for every $ξ_1, ξ_2 ∈ ℝ$, $0 < ξ_0 ≤ a(ξ_1, ξ_2) ≤ h(ξ)$ (a.e. $s ∈ (0, 1)$), with $h ∈ L^1_{per}(0, 1)$. The usual Tartar-Fonseca assumption on $W$ used in [5] and [1] (cf. also classical references [2], [4]), namely: there exists $c_1 > 0$, and $R_0 > 1$ such that for every $|ξ| > R_0$ there holds $W(ξ) ≥ c_1|ξ|$ (which, in the case $a = 0$, immediately yields equi-coercivity of $ε^{-1}I_0^ε$ on $W^{1,1}(0, 1)$, see [3]), is replaced by non-standard growth condition on $W$: there exist $q ∈ ℝ$, $R_0 > 1$ and $C_2 > 0$, $C_1 > 0$ such that for every $|ξ| ≥ R_0$ there holds $C_0 ≤ W(ξ) ≤ C_2|ξ|$. If $q ≤ 0$ and $a > 0$ we were able to show that arbitrary finite-energy sequence of $ε^{-2/3}I_0^ε$ can be well-approximated by 1-Lipschitz and 1-periodic FE sequence in the sense of theorems below. The proofs rely on two essential ingredients: on Leoni’s results in [3] in the case $a = 0$ (based on an application of area formula, an argument which we adapt to case $a > 0$), and on deployment of the blow-up technique developed in [1], which has been successfully applied to similar functionals in papers [6]- [18]. As such, our results are: (1) an extension of the independence of boundary conditions property obtained in [18] and briefly described in section 6 in [1]; (2) the first step in obtaining results similar to Theorem 1.1 and Corollary 1.2 in [5], where the case of constant function $a$ was considered, and where periodic boundary conditions where used on the level of minimizers of $I_0^ε$. In order to capture asymptotic equivalence of arbitrary FE sequence and the corresponding approximation of such FE sequences, we use the setting of Young measures on micro-patterns (or $K$-valued Young measures) developed in [1]. The set of all $K$-valued Young measures is denoted by $Y.M((0, 1); K)$ and it is defined as the dual of $L^1((0, 1); C(K))$, where $K$ is the compact metric space of all Borel measurable functions from $ℝ$ to $[−∞, +∞]$, endowed with the metric according to formula (5.1) in section 5 in [1]. By $Φ$ we denote the metric on $Y.M((0, 1); K)$ which induces the usual weak-star topology. If $v ∈ K$ is given, we define $ε$-blow-up of $v, R^εv : (0, 1) → K$, by $R^εv(τ) := ε^{-1/3}v(s + ε^{1/3}τ)$, where $s ∈ (0, 1)$. We also define $δ_{R^εv}(s) ∈ Y.M((0, 1); K)$ by $δ_{R^εv}(s) := δ_{R^εv,v}, s ∈ (0, 1)$, where $δ_{R^εv,v}$ is the usual Dirac mass centered at $R^εv$. By Lip($v$) we denote the Lipschitz constant of $v ∈ K$. We introduce the notation $T_εv(s) := 10 ∫_0^1 v(1/3(|ξ|)dξ + v(0)), s ∈ (0, 1)$, where $δ > 0$ and $v_δ^{(1)} := max{min{v′, 1 + δ}, −1 − δ}$ is the standard truncation of $|v′|$ at level $1 + δ$.

2 Main results

In this section we state (without the proof) currently available results regarding approximation of arbitrary FE sequences by 1-Lipschitz and 1-periodic ones. We recall that a sequence $(v_ε)$ is said to be FE sequence of $(ε^{-2/3}I_0^ε)$ if there holds $lim sup_{ε → 0} ε^{-2/3}I_0^ε(v_ε) < +∞$.

Theorem 2.1 (Approximation by 1-Lipschitz FE sequence)

Let $q ≤ 0$. Then for every FE sequence $(v_ε)$ for $(ε^{-2/3}I_0^ε)$ in $H^2(0, 1)$ and every $1 > δ > 0$ there holds:

(i) $lim_{ε → 0} Φ(δ_{R^εv(T_εvε)}; δ_{R^εv,v}) = 0$,
We do not know if it is possible to extend assertions of Theorem 2.1, or Theorem 2.2, to the case
$q ≥ 0$ (i.e., $W ∈ C_0(R)$). Based on similar conclusion of Leoni in [3], we conjecture that there exists some critical value $q_c ≥ 0$ such that Theorem 2.2 (or some analogous assertion) holds for every $q ∈ (−∞, q_c)$.

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References